• LET $g(x) = \sum_{n=0}^{\infty} \frac{1}{3^n} x^n$. COMPUTE THE PRODUCT $g(x) \cdot g(x)$.

$$g(x) = \sum_{n=0}^{\infty} \left(\frac{x}{3}\right)^n = 1 + \frac{x}{3} + \frac{x^2}{3^2} + \frac{x^3}{3^3} + \frac{x^4}{3^4} + \cdots$$

SO LET'S JUST MULTIPLY AND SEE WHAT HAPPENS.

$$g(x) \cdot g(x) = \left(1 + \frac{x}{3} + \frac{x^2}{3^2} + \frac{x^3}{3^3} + \frac{x^4}{3^4} + \cdots\right)\left(1 + \frac{x}{3} + \frac{x^2}{3^2} + \frac{x^3}{3^3} + \frac{x^4}{3^4} + \cdots\right)$$

$$= 1 + \left(\frac{x}{3} + \frac{x}{3}\right) + \left(\frac{x^2}{9} + \frac{x^2}{9} + \frac{x^2}{9}\right) + \left(\frac{x^3}{3^3} + \frac{x^3}{3^3} + \frac{x^3}{3^3} + \frac{x^3}{3^3}\right) + \cdots$$

$$= 1 + \frac{2x}{3} + \frac{3x^2}{9} + \frac{4x^3}{3^3} + \frac{5x^4}{3^4} + \cdots$$

$$= \sum_{n=1}^{\infty} \frac{n x^{n-1}}{3^{n-1}}$$

WE COULD ALSO DO THIS BY NOTICING THAT

$g(x) = \frac{1}{1 - x/3} = \frac{3}{3-x}$ AND $[g(x)]^2 = \frac{9}{(3-x)^2} = 9 \cdot \frac{d}{dx}\left(\frac{1}{3-x}\right)$

WE'D GET THE SAME ANSWER ...

• LET $f(x) = \frac{x}{1-x^2}$. WRITE A POWER SERIES FOR f

START WITH THE GEOMETRIC SERIES $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ AND

SUBSTITUTE $x \to x^2$ TO GET $\frac{1}{1-x^2} = \sum_{x=0}^{\infty} x^{2n}$

THEN MULTIPLY BY x TO GET $\frac{x}{1-x^2} = \sum_{x=0}^{\infty} x^{2n+1}$