EXERCISE ONE Let $\{a_n\}_{n=1}^{+\infty}$ be a sequence of real numbers whose terms are

$$2, -\sqrt{2}, 1, -\frac{\sqrt{2}}{2}, \frac{1}{2}, \cdots$$

Assume that the pattern continues. Prove that the series $\sum_{n=1}^{+\infty} a_n$ converges and give its sum.

The sequence $\{a_n\}$ is a geometric series with ratio $-\frac{1}{\sqrt{2}}$. As this ratio has absolute value < 1, the series is convergent. An explicit formula for the sequence $\{a_n\}$ is $a_n = 2(-\frac{1}{\sqrt{2}})^{n-1}$. By the formula we learned in class the sum is $\frac{2}{1-(-\frac{1}{\sqrt{2}})} = \frac{2}{1+\frac{1}{\sqrt{2}}} = \frac{2\sqrt{2}}{\sqrt{2}+1}$.

EXERCISE TWO Consider the series $\sum_{n=1}^{+\infty} \frac{n^2+1}{127n^2+1}$. Does it converge? Please justify your answer.

Consider the term sequence $\{\frac{n^2+1}{127n^2+1}\}$. We have $\lim \frac{n^2+1}{127n^2+1} = \lim \frac{1+\frac{1}{n^2}}{127+\frac{1}{n^2}} = \frac{1}{127}$. As it is not 0, the series diverges by Divergence Test.