EXERCISE ONE Compute the following two complex numbers. You need to write the answer in the form $a + bi$ where a, b are explicit real numbers. You cannot use $+, -, \times, /, \sin, \cos, e^{\theta}$ in the final expression.

1. $i^{99} + i^{88} - (-i)^{105} + 5i$;
2. $\frac{(1-i)(2-i)}{3+\sqrt{2}i}$.

We have $i^0 = 1, i^1 = i, i^2 = -1, i^3 = -i, i^4 = 1 = i^0$. The pattern is 4-periodic. Thus for any integer k, we have $i^{4k} = 1, i^{4k+1} = i, i^{4k+2} = -1, i^{4k+3} = -i$. As $99 = 4 \times 24 + 3, 88 = 4 \times 22, 105 = 4 \times 26 + 1$, we have

$$\begin{aligned} i^{99} + i^{88} - (-i)^{105} + 5i &= -i + 1 - (-1)^{105} i + 5i \\ &= -i + 1 + i + 5i \\ &= 1 + 5i \end{aligned}$$

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\begin{aligned}
\frac{(1-i)(2-i)}{3+\sqrt{2}i} &= \frac{1-3i}{3+\sqrt{2}i} \\
&= \frac{(1-3i)(3-\sqrt{2}i)}{(3+\sqrt{2}i)(3-\sqrt{2}i)} \\
&= \frac{3-3\sqrt{2}-(9+\sqrt{2})i}{11}
\end{aligned}
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