

EXERCISE ONE Let $\{a_n\}_{n=1}^{+\infty}$ be a sequence of real numbers defined by the following recurrence relation:

- $a_1 = 3$;
- $a_{n+1} = \frac{a_n}{2} + \frac{1}{2}$ for all n .

Let $\{b_n\}_{n=1}^{+\infty}$ be a sequence such that $b_n = a_n - 1$ for all n . Prove that b_n is a geometric sequence and find an explicit formula for $\{b_n\}$.

EXERCISE TWO Consider the sequence $\left\{ \frac{126e^{\frac{1}{n}} + 1}{\cos\left(\frac{1}{n^2}\right)} \right\}$. Does it converge? If yes what is the limit?