PRINT your name: SOLUTIONS

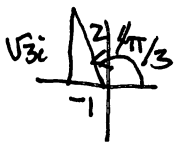
Answer each question below completely. You must fully justify your answers to get credit. Even a correct answer with no justification is wrong.

1. Simplify $\dfrac{-1+i\sqrt{3}}{3+i\sqrt{3}}$. Give your answer in the form $a+bi$.
   If either a or b is 0, feel free to omit that part of the answer.

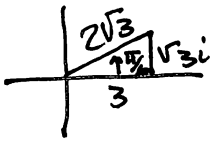
ALGEBRAICALLY: $\left(\dfrac{-1+i\sqrt{3}}{3+i\sqrt{3}}\right)\left(\dfrac{3-i\sqrt{3}}{3-i\sqrt{3}}\right) = \dfrac{-3+i\sqrt{3}+3i\sqrt{3}-i^2\cdot 3}{12}$

$= \dfrac{4i\sqrt{3}}{12} = \boxed{i\,\dfrac{\sqrt{3}}{3} = \dfrac{i}{\sqrt{3}}}$

USING EXPONENTIAL:



$-1+i\sqrt{3} = 2e^{\frac{4\pi}{6}i}$,

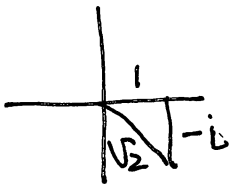


$3+i\sqrt{3} = 2\sqrt{3}e^{\pi/6\,i}$

so $\dfrac{-1+i\sqrt{3}}{3+i\sqrt{3}} = \dfrac{2e^{4\pi i/6}}{2\sqrt{3}e^{\pi/6\,i}} = \dfrac{e^{\pi/2\,i}}{\sqrt{3}} = \boxed{\dfrac{i}{\sqrt{3}} = \dfrac{i\sqrt{3}}{3}}$

2. Write $(1-i)^8$ in the form $a+bi$.

USING EXPONENTIAL



$1-i = e^{-\pi/4\,i}$, so $(1-i)^8 = \left(\sqrt{2}e^{-\frac{\pi}{4}i}\right)^8 = (\sqrt{2})^8 e^{8\cdot -2\pi i}$

$= (\sqrt{2})^8 = \boxed{16}$

ALGEBRA: $(1-i)^8 = \left((1-i)^2\right)^4 = (0-2i)^4 = (-2)^4 i^4$

$= \boxed{16}$