PRINT your name: SOLNS.

The chemicals methyl bromide (CH_3Br) and sodium thiosulphate ($Na_2S_2O_3$) react to produce Br^- and $NaCH_3S_2O$. The total amount of sodium thiosulfate $S(t)$ at time t is well modeled by the logistic equation $S'(t) = rS(t)(S(t) - K)$. The constant r can be taken to be $-1/120$, and K depends on the relative concentration of the two chemicals. For the purposes of this question, let $K = 180$.

Since some of you may have memorized the solution to the logistic equation, I might as well tell you that $S(t) = \dfrac{K}{1 + CKe^{rKt}}$ for some constant C.

1. If the initial amount of sodium thiosulphate is 300 grams, what will be the amount after a long time?

$S(t) = \dfrac{180}{1 + 180Ce^{-180/120 t}} = \dfrac{180}{1 + C_1 e^{-3t/2}}$

$\lim_{t\to\infty} S(t) = \dfrac{180}{1 + \lim_{t\to\infty} C_1 e^{-3t/2}} = \boxed{180}$

SO CLOSE TO 180g AFTER A LONG TIME.

2. If the initial amount of sodium thiosulphate is 30 grams, at what time t will there be 60 grams?

Please leave your answer as an exact number. Do not approximate e, fractions, logarithms, π and so on.

TO FIND C_1, SOLVE $S(0) = 30 = \dfrac{180}{1 + C_1 e^{0}}$, i.e.

$1 + C_1 = \dfrac{180}{30} = 6$

$\boxed{C_1 = 5}$.

SO $S(t) = \dfrac{180}{1 + 5e^{-3t/2}}$

NOW FIND t SO THAT $S(t) = 60$:

$60 = \dfrac{180}{1 + 5e^{-3t/2}}$

$\Rightarrow 1 + 5e^{-3t/2} = \dfrac{180}{60} = 3$

$5e^{-3t/2} = 2$

$e^{-3t/2} = \dfrac{2}{5}$

TAKE LOG, $-3t/2 = \ln\left(\dfrac{2}{5}\right) \Rightarrow t = -\dfrac{2}{3}\ln\left(\dfrac{2}{5}\right) = \boxed{\dfrac{2}{3}\ln\left(\dfrac{5}{2}\right)}$

EITHER IS OK