PRINT your name:

1. The power series $\sum_{n=0}^{\infty} \frac{(x/2)^{2n}}{n^2+1} = 1 + \frac{x^2}{4 \cdot 2} + \frac{x^4}{16 \cdot 5} + \dots$ has a radius of convergence equal to 2. Find its interval of convergence. Fully justify your answer.

SINCE THE RADIUS OF CONVERGENCE IS KNOWN TO BE 2 (WE COULD USE THE RATIO TEST TO GET THIS), WE KNOW THE SERIES CONVERGES FOR $-2<x<2$. WE MUST CHECK $x=2$ AND $x=-2$.

$\underline{x=2}$ SERIES IS $\sum_{n=0}^{\infty} \frac{1}{1+n^2}$. WE CAN DETERMINE CONVERGENCE WITH THE INTEGRAL TEST OR COMPARISON. LETS COMPARE TO THE CONVERGENT P-SERIES WITH $p=2$: $\frac{1}{1+n^2} < \frac{1}{n^2}$, SO SINCE $\sum_1 \frac{1}{n^2}$ CONVERGES, SO DOES $\sum_1 \frac{1}{n^2+1}$.

$\underline{x=-2}$ SERIES IS $\sum_1 \frac{(-1)^n}{n^2+1}$. SINCE $\sum_1 \frac{1}{n^2+1}$ CONVERGES AND $\left|\frac{(-1)^n}{1+n^2}\right| = \frac{1}{1+n^2}$, SERIES CONVERGES. OR USE THE ALT. SERIES TEST.

$\boxed{\text{INTERVAL IS } x \in [-2, 2]}$

2. Does the infinite series $\sum_{n=0}^{\infty} \frac{3n^2-1}{n^3-n+3}$ converge or diverge? Fully justify your answer.

THIS DIVERGES. YOU COULD USE THE INTEGRAL TEST, SINCE $\int \frac{3x^2-1}{x^3-n+3}dx = \int \frac{du}{u}$ WITH $u = x^3-x+3$.

BUT I'D RATHER USE LIMIT COMPARISON WITH $\sum \frac{1}{n}$ (DIVERGENT).

$$\lim_{n\to\infty} \left(\frac{3n^2-1}{n^3-n+3} \cdot \frac{1}{1/n} \right) = \lim_{n\to\infty} \left(\frac{3n^3-1}{n^3-n+3} \right) = 3$$

SINCE THE RATIO IS NOT 0, $\sum_1 \frac{3n^2-1}{n^3-n+3}$ AND $\sum_1 \frac{1}{n}$ DO THE SAME, THAT IS $\boxed{\text{DIVERGE}}$

(YOU COULD ALSO USE DIRECT COMPARISON WITH $\sum_1 \frac{3}{n}$)