PRINT your name:

Recall that the power series $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ for certain values of x.

1. Write a power series that converges to $\frac{4x}{2+x^2}$ for some values of x. Make sure you explain how you arrived at your answer.

$$\frac{4x}{2+x^2} = 4x\left(\frac{1/2}{1-\left(\frac{-x^2}{2}\right)}\right) = 2x\left(\frac{1}{1-\left(\frac{-x^2}{2}\right)}\right)$$

$$= 2x\left(\sum_{n=0}^{\infty}\left(\frac{-x^2}{2}\right)^n\right) = 2x\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{2^n} = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{2^{n-1}}$$

$$= 2x - x^3 + \frac{x^5}{2} - \frac{x^7}{2^2} + \frac{x^9}{3} - \cdots$$

2. What is the center and radius of convergence of the series above? That is, the largest open interval on which the series converges is

$$-\sqrt{2} < x - 0 < \sqrt{2}$$

Fully justify how you got this result. (CENTER IS 0, RADIUS IS $\sqrt{2}$)

TO SEE THIS, YOU COULD USE THE RATIO TEST, BUT ALSO, SINCE THE ORIGINAL GEOMETRIC SERIES CONVERGES FOR $|x|<1$ AND WE SUBSTITUTED $x \to \frac{-x^2}{2}$ SO THE NEW SERIES CONVERGES FOR $\left|\frac{-x^2}{2}\right| < 1$, THAT IS $x^2 < 2$

IE $\boxed{-\sqrt{2} < x < \sqrt{2}}$