PRINT your name:

Answer each question completely. You must fully justify your answers to get credit. Even a correct answer with no justification is wrong.

1. Does the series $\sum_{n=1}^{\infty}\left(\frac{1}{n^2}-\frac{1}{(n+3)^2}\right)$ converge or diverge? If it converges, find the sum. If it diverges, say so and justify your answer fully.

THIS IS A TELESCOPING SERIES. LET'S LOOK AT A FEW TERMS.

$$\sum_{n=1}^{\infty}\left(\frac{1}{n^2}-\frac{1}{(n+3)^2}\right) = \left(1 - \cancel{\frac{1}{4^2}}\right) + \left(\frac{1}{2^2} - \cancel{\frac{1}{5^2}}\right) + \left(\frac{1}{3^2} - \cancel{\frac{1}{6^2}}\right) + \left(\cancel{\frac{1}{4^2}} - \cancel{\frac{1}{7^2}}\right)$$
$$+ \left(\cancel{\frac{1}{5^2}} - \cancel{\frac{1}{8^2}}\right) + \left(\cancel{\frac{1}{6^2}} - \cancel{\frac{1}{9^2}}\right) + \cancel{\cdots}$$
$$= 1 + \frac{1}{4} + \frac{1}{9}$$

(MORE FORMALLY, WE HAVE $S_k = \sum_{n=1}^{k}\left(\frac{1}{n^2}-\frac{1}{(n+3)^2}\right) = 1 + \frac{1}{4} + \frac{1}{9} - \frac{1}{(k+1)^2} - \frac{1}{(k+2)^2} - \frac{1}{(k+3)^2}$

THEN $\lim_{k\to\infty} S_k = \lim_{k\to\infty}\left(1 + \frac{1}{4} + \frac{1}{9} - \frac{1}{(k+1)^2} - \frac{1}{(k+2)^2} - \frac{1}{(k+3)^2}\right) = 1 + \frac{1}{4} + \frac{1}{9}$)

2. Consider the series $\sum_{n=0}^{\infty}\frac{(-1)^n\pi^{n+2}}{6^n}$. If it converges, find the sum. If it diverges, explain why.

THIS CAN BE WRITTEN AS A GEOMETRIC SERIES

$$\sum_{n=0}^{\infty}\frac{(-1)^n\pi^{n+2}}{6^n} = \pi^2\sum_{n=0}^{\infty}\frac{(-1)^n(\pi)^n}{6^n}$$
$$= \pi^2\sum_{n=0}^{\infty}\left(\frac{-\pi}{6}\right)^n$$
$$= \frac{\pi^2}{1+\pi/6} = \frac{6\pi^2}{6+\pi}$$