PRINT your name:

Answer each question completely. You must fully justify your answers to get credit. Even a correct answer with no justification is wrong.

1. Let $\{a_n\}_{n=1}^{\infty}$ satisfy $a_1 = 6, \quad a_{n+1} = \dfrac{2a_n}{3}$ for $n > 1$.

   Write the first four terms of the sequence and then give a formula for a_n that does not depend explicitly on previous terms.

$a_1 = 6$, $a_2 = \frac{2}{3}\cdot 6$, $a_3 = \frac{2}{3}\left(\frac{2}{3}\cdot 6\right) = \left(\frac{2}{3}\right)^2 \cdot 6$

$a_4 = \left(\frac{2}{3}\right)^3 \cdot 6$ ETC.

$$a_n = \left(\frac{2}{3}\right)^{n-1} \cdot 6$$

2. Let $b_n = \dfrac{\sqrt{n^2+5}}{n^2} + \cos\left(\dfrac{\pi}{n}\right)$.

   Does the sequence $\{b_n\}_{n=1}^{\infty}$ converge or diverge?
   If it converges, **calculate the limit**. If it diverges, explain why.

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{\sqrt{n^2+5}}{n^2} + \lim_{n\to\infty} \cos\left(\frac{\pi}{n}\right)$$

$$= \lim_{n\to\infty} \sqrt{\frac{n^2}{n^4} + \frac{5}{n^4}} + \cos\left(\lim_{n\to\infty} \frac{\pi}{n}\right)$$

$$= 0 + \cos(0)$$

$$= 1$$

(SINCE THE LIMIT EXISTS, THE SEQUENCE CONVERGES)