MATH 127 Final Exam

Name: ______________________ **ID:** ______________________ **Lecture:** ________

Part I:

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	10	10	10	10	10	10	10	10	10	10	100
Score:											

Part II:

Question:	11	12	13	14	15	16	17	Total
Points:	15	20	15	25	15	15	15	120
Score:								

Please circle your lecture:

- Lecture 1 (Babak Modami, MWF 10:00am)
- Lecture 2 (Christian Schnell, TTh 2:30pm)
- Lecture 4 (Timothy Ryan, TTh 4:00pm)

Instructions:

1. You have 2 hours and 30 minutes to complete this exam.
2. There are 10 short problems plus 7 longer ones, printed on 11 pages including this cover sheet. Make sure that you have all of them. Do the shorter problems in Part I first.
3. Do all of your work in this exam booklet, and cross out anything that the grader should ignore. You may use the backs of pages, but indicate clearly what is where if you expect someone to look at it.
4. You must give a correct justification of all answers to receive credit. Leave all answers in exact form: do not approximate π, square roots, fractions, and so on.
5. Books, calculators and other electronic devices, extra papers, or discussions with friends are **not permitted**. If you have a cellphone, it must be **switched off** and put away.

Part I: Do These First!

Name: ____________________

10 pts 1. Write the infinite decimal $0.121212\ldots$ as a fraction.

10 pts 2. Write a power series for the function e^{-x^2}.

10 pts 3. The sequence $a_n = 1 + (-1)^n$ converges / diverges (circle one).
Justify:

10 pts 4. Find the solution to the initial-value problem $\dfrac{dy}{dt} = -2y$, $y(0) = 5$.

Part I: Do These First!

Name: ______________________

10 pts   5. Solve the differential equation $\dfrac{dy}{dx} = xe^{-y}$.

10 pts   6. Find the value of the infinite sum $1 + \dfrac{1}{3} + \dfrac{1}{9} + \dfrac{1}{27} + \dfrac{1}{81} + \cdots$.

10 pts   7. The series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges / diverges (circle one) by the ______________________ test.

Justify:

10 pts 8. The series $\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$ converges / diverges (circle one) by the ____________________ test.
Justify:

10 pts 9. Determine all values of r for which the function $y = e^{rx}$ solves the differential equation $y'' + 3y' + y = 0$.

10 pts 10. Iodine-131 has a half life of approximately 8 days. If we initially have 1 gram of this substance, then how much iodine-131 will be left after 40 days?

15 pts 11. Write the first four terms in the Taylor series expansion of $\ln x$ around the point $a = 5$.

20 pts 12. Find all values of x for which the power series $\sum_{n=1}^{\infty} \frac{(2x-1)^n}{n^2}$ converges. Do not forget to check the endpoints!

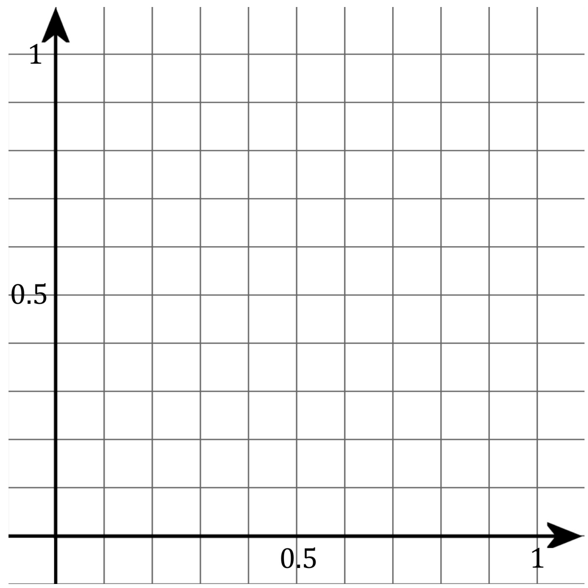
Part II: Do These After Part I!

Name: ____________________

15 pts 13. Let $y = y(x)$ be the solution of the initial value problem $y' = x - y$, $y(0) = 1$.

(a) Use Euler's method with step size 0.2 to estimate $y(0.4)$.

(b) In the coordinate system below, plot the three points (x_0, y_0), (x_1, y_1), and (x_2, y_2) that you get from Euler's method.



Part II: Do These After Part I! **Name:** ______________________

25 pts 14. In this problem, we use the following two differential equations to model the populations of rabbits (R) and wolves (W).

$$\frac{dR}{dt} = 0.08R(1 - 0.0002R) - 0.001RW$$
$$\frac{dW}{dt} = -0.02W + 0.00002RW$$

(a) According to these equations, what happens to the rabbit population in the absence of wolves?

(b) Find all the equilibrium solutions and explain their significance.

NOTE THAT THIS TOPIC WAS NOT COVERED IN MAT127 DURING SPRING 2021, SO YOU ARE NOT RESPOSIBLE FOR IT.

15 pts 15. The value of the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is known to be $\frac{\pi^2}{6}$. Suppose we approximate $\frac{\pi^2}{6}$ by adding together the first n terms of the series. How big should n be, in order for the error in our approximation to be at most 0.001?

Part II: Do These After Part I!   **Name:** ____________________

15 pts 16. The initial-value problem

$$y'' = xy' + y, \qquad y(0) = 2, \quad y'(0) = -1$$

has a power series solution $y = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + \cdots$.

Determine the values of the first five coefficients c_0, c_1, c_2, c_3, c_4.

Part II: Do These After Part I!

Name: ____________________________

15 pts 17. Determine all values of x for which the infinite series $\sum_{n=1}^{\infty} (\ln x)^n$ is convergent.

Be sure to justify your answer.