

Final Exam
MAT 127
December 17, 2018

Name: <small>(please print)</small>	ID #:
Your section:	(see list below)

Lecture 1	MWF 10:00am-10:53am	Chuanhao Wei
Lecture 2	MF 1:00 pm – 2:20 pm	Jingrui Cheng
Lecture 3	TuTh 10:00 am – 11:20 am	Sabyasachi Mukherjee
Lecture 4	TuTh 5:30 pm – 6:50 pm	Babak Modami

No notes, books or calculators.

You must show your reasoning, not just the answer. Answers without justification will get only partial credit. Your solutions should be written so that the grader is able to follow your reasoning and computations.

Please cross out anything that is not part of your solution — e.g., some preliminary computations that you didn't need. Everything not crossed out will be considered part of your solution and graded.

When computing numerical answers, please do not replace algebraic expressions and constants such as $\sqrt{2}$ or π by approximate values — instead, leave $\sqrt{2}$ in your final answer.

	1	2	3	4	5	6	7	Total
	15pt	15pt	15pt	25pt	10pt	10pt	10pt	100pts
<i>Grade</i>								

1. (5+5+5 pts)

For each of the following sequences, determine whether it converges. If it converges, find the limit.

(a)

$$a_n = \frac{\cos(n\pi/8)(2^n + 1)}{5^n}$$

(b)

$$a_n = \frac{n^3 + 2n + 4}{2n(n^2 + 1)}$$

(c)

$$a_n = \frac{\ln(n) + 1}{n^{1/3}}$$

2. (5+5+5 pts)

For each of the series below, determine whether the series converges. You do not need to find the sum of the series.

(a)

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2 + 2n - 2}$$

(b)

$$\sum_{n=2}^{\infty} \frac{1}{n (\ln(n))^2}$$

(c) Determine whether the following series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{\cos(n\pi/8)(2^n + 1)}{5^n}$$

3. (5+5+5 pts)

Let

$$f(x) = x^2 \ln\left(1 + \frac{x^2}{2}\right).$$

(a) Write the Taylor series for $f(x)$, centered at 0.

(b) Find the radius of convergence of this series.

(c) Write $\int_0^1 f(x)dx$ as a series.

4. (10+15 pts)

(a) Solve:

$$y' = x^2y + x^2, \quad y(0) = 1.$$

(b) Solve:

$$y'' + 2y' + y = 0, \quad y(0) = 2, \quad y'(0) = 2.$$

5. (10 pts)

Use Eulers method with step size 0.1 to estimate $y(0.3)$ where $y(x)$ satisfies:

$$y' = 2x + y, \quad y(0) = 1.$$

6. (10 pts)

A radioactive material has a half-life of 100 years. How long does it take to decay to one-third of its original mass?

Or

A certain small country has \$10 billion in paper currency in circulation, and each day \$50 million comes into the country's banks. The government decides to introduce new currency by having the banks replace old bills with new ones whenever old currency comes into the banks. Let $x(t)$ denote the amount of new currency in circulation at time t , with $x(0) = 0$. How long will it take for the new bills to account for 90% of the currency in circulation?

7. (10 pts)

A population of bees in a particular region satisfies the logistic equation with carrying capacity 10000. Suppose that there are 1000 bees initially and 2000 bees after 2 years. How many bees are there after 4 years?