Quiz 9 - Solutions

1) $y = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5 + \cdots$

$\Rightarrow y' = c_1 + 2c_2 x + 3c_3 x^2 + 4c_4 x^3 + 5c_5 x^4 + \cdots$

$\Rightarrow y'' = 2c_2 + 3\cdot 2 c_3 x + 4\cdot 3 c_4 x^2 + 5\cdot 4 c_5 x^3 + \cdots$

Since $y(0) = 1 \Rightarrow \boxed{c_0 = 1}$ and $y'(0) = -1 \Rightarrow \boxed{c_1 = -1}$

$$y'' = 2 + y' \Rightarrow 2c_2 + 3\cdot 2 c_3 x + 4\cdot 3 c_4 x^2 + 5\cdot 4 c_5 x^3 + \cdots$$
$$= 2 + c_1 + 2c_2 x + 3c_3 x^2 + 4c_4 x^3 + \cdots$$

$$\Rightarrow \begin{cases} 2c_2 = 2 + c_1 \\ 3\cdot 2 c_3 = 2c_2 \\ 4\cdot 3\, c_4 = 3c_3 \\ 5\cdot 4\, c_5 = 4c_4 \\ \vdots \end{cases} \Rightarrow \begin{cases} c_0 = 1 \\ c_1 = -1 \\ c_2 = \dfrac{2+c_1}{2} = \dfrac{2-1}{2} = \dfrac{1}{2} = \dfrac{1}{2!} \\ c_3 = \dfrac{\cancel{2}c_2}{3\cdot \cancel{2}} = \dfrac{c_2}{3} = \dfrac{1}{3\cdot 2!} = \dfrac{1}{3!} \\ c_4 = \dfrac{\cancel{3}c_3}{4\cdot \cancel{3}} = \dfrac{c_3}{4} = \dfrac{1}{4\cdot 3!} = \dfrac{1}{4!} \\ c_5 = \dfrac{\cancel{4}c_4}{5\cdot \cancel{4}} = \dfrac{c_4}{5} = \dfrac{1}{5\cdot 4!} = \dfrac{1}{5!} \\ \vdots \\ c_n = \dfrac{1}{n!} \\ \vdots \end{cases}$$

So, $y = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5 + \cdots$

$$\Rightarrow \boxed{y = 1 - x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots}$$

2) Since $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots$

we observe that

$$y = 1 - x + (e^x - 1 - x) = \cancel{1} - x + e^x - \cancel{1} - x = e^x - 2x$$

$$\Rightarrow \boxed{y = e^x - 2x}$$