Quiz 8

1) $\frac{dT}{dt} = k(T-20)$

$T-20=0 \iff T=20 \rightarrow$ constant solution

If $T \neq 20$ then $\frac{dT}{dt} = k(T-20) \Rightarrow \frac{dT}{T-20} = k\,dt \Rightarrow$

$\int \frac{dT}{T-20} = \int k\,dt \Rightarrow \ln|T-20| = kt + C \Rightarrow$

$|T-20| = e^{kt} \cdot C \Rightarrow T-20 = Ce^{kt} \Rightarrow T = Ce^{kt} + 20$

Initial condition: $T(0)=40 \Rightarrow Ce^{0}+20=40$

$\Rightarrow C=40-20 \Rightarrow \boxed{C=20}$. So, the answer is

$$T(t)=20+20e^{kt}$$

2) $T(1)=30 \Rightarrow 20+20e^{k}=30 \Rightarrow$

$20e^{k}=10 \Rightarrow e^{k}=1/2 \Rightarrow k=\ln(1/2)$