Quiz 11

1) $y'' + 3y' + y = -2y' - 3y \Rightarrow y'' + 5y' + 4y = 0.$

Characteristic equation: $r^2 + 5r + 4 = 0.$

Deteminant $D = 5^2 - 4 \cdot 4 = 25 - 16 = 9 > 0 \Rightarrow$ there are 2 different real roots

$$r_1 = \frac{-5 + \sqrt{9}}{2} = \frac{-5+3}{2} = \frac{-2}{2} = -1 \quad \text{and}$$

$$r_2 = \frac{-5 - \sqrt{9}}{2} = \frac{-5-3}{2} = \frac{-8}{2} = -4$$

So, the general solution is $y = C_1 e^{-x} + C_2 e^{-4x}$, where $C_1, C_2 \in \mathbb{R}$

2) We have $y' = -C_1 e^{-x} - 4C_2 e^{-4x}$. By the initial conditions we get

$$\left.\begin{aligned} y(0) = 0 &\Rightarrow C_1 + C_2 = 0 \\ y'(0) = 2 &\Rightarrow -C_1 - 4C_2 = 2 \end{aligned}\right\} \overset{(+)}{\Rightarrow} -3C_2 = 2 \Rightarrow \boxed{C_2 = -2/3}$$

and $\boxed{C_1 = -C_2 = 2/3}$

So, $y = \frac{2}{3} e^{-x} - \frac{2}{3} e^{-4x}$