Name: ____________________

ID#: ____________________

Test # 2(practice)

MAT 127 Spring 2005

Directions: There are 4 questions. You have 1 hour. For credit, you must show all your work, using the backs of the pages if necessary. You may not use a calculator.

1. ____/25 2. ____/25 3. ____/25 4. ____/25

Total Score. ____/100

1. Let k and K be positive constansts. Assume the function $y(t)$ satisfies the initial value problem

$$\frac{dy}{dt} = ky\left(1 - \frac{y}{K}\right) \qquad y(0) = 10.$$

(a) If $K = 100$ and $y(\ln(9/2)) = 50$ find the solution to the initial value problem.
The answer is

$$y(t) = \frac{100}{1 + 9e^{-t}}.$$

(b) If $k = 1/2$ and $y(2 \cdot \ln(6/5)) = 8$ find the solution to the initial value problem.
The answer is

$$y(t) = \frac{4}{1 - (3/5)e^{-t/2}}$$

(c) Assume the conditions in part (b). When will $y(t) = 6$? When will $y(t) = 4$? Explain your answer to the latter question.
Tne answer to the first question is $t = 2 \cdot \ln(9/5)$. The answer to the second question is "never"; the reason being that the carrying capacity is $K = 4$ so that it never attains that number (though it does approach it as t gets big).

2. Consider the sequence whose n^{th} term is $a_n = n/(n+4)$.

   (a) Show that the sequence is increasing.
   Set up the inequality you want to show:

$$(n+1)/((n+1)+4) > n/(n+4)$$

   Now use algebra to show that this inequality is true if and only if the inequality $4 > 0$ is true; then you're done.

   (b) Assume the sequence converges and compute its limit.
   The sequence converges to 1.

3. Compute the limits of the following convergent sequences:

   (a) $\left\{ \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}{(2n)^n} \right\}$
   The sequence converges to zero. You'll want to use the "squeeze theorm" to show this.

   (b) $\{\ln(n+1) - \ln(n)\}$
   The sequence converges to zero. Use the rules for logarithms and L'Hospital's rule so show this.

4. Compute the limit of the following convergent sequence: (Hint: Use the fact that if $\lim_{n\to\infty} a_n = L$ then, obviously, $\lim_{n\to\infty} a_{n+1} = L$, as well).

$$\left\{\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, ...\right\}$$

The sequence converges to 2.