Name: ____________________

ID#: ____________________

Final Exam

MAT 127 Spring 2005

Directions: There are 8 questions. You have until 1:30 PM (150 minutes). For credit, you must show all your work, using the backs of the pages if necessary. You may not use a calculator.

1. ____/10 2. ____/10 3. ____/10 4. ____/10 5. ____/15 6. ____/15 7. ____/15 8. ____/15

Total Score. ____/100

1. A function $y(t)$ satisfies the differential equation

$$\frac{dy}{dt} = y^2 - y - 6.$$

(a) What are the constant solutions of the equation?

(b) For what values of y is y increasing?

(c) For what values of y is y decreasing?

2. Solve the following initial value problems. (Hint: they are both seperable.)
   (a) $\frac{dx}{dt} = -2x^2 t$, $x(0) = 1/3$
   (b) $\frac{dy}{dt} = y \cos t$, $y(\pi) = 1$

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3. Assume a contagious disease spreads at a rate proportional to the number of infected people. Initially there are 10 people infected and after 1 month there are 100 people infected.
   (a) Find an expression for the number of infected people after t months.
   (b) When will there be 1000 infected people?

4. Compute the limits of the following convergent sequences:

   (a) $\left\{\frac{\sin 5n}{2n}\right\}$

   (b) $\left\{\frac{n}{(\ln n)^2}\right\}$

5. Determine whether the series is convergent or divergent. State which test you're using.

   (a) $\sum_{n=1}^{\infty} \frac{3n}{n^3+4}$

   (b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+2}}$

6. Compute the following sums.
   (a) $\sum_{n=0}^{\infty} \frac{3^{2n+1}}{10^n}$
   (b) $\sum_{n=1}^{\infty} [\sin(1/n) - \sin(1/(n+1))]$

7. Compute and write out the following series. If applicable, you can use the table of Maclaurin series provided with your exam:
   (a) The Maclaurin series for $f(x) = x^3 e^{x^3}$.
   (b) The Taylor series, centered around $a = 1$, for $f(x) = x^3$.

8. Consider the power series

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n3^n}.$$

(a) Write the "center" of this power series.

(b) Find the open interval of absolute convergence.

(c) Determine whether the series converges or diverges at each of the interval's endpoints.

Here are some Maclaurin series from the text:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad \text{for} \quad |x| < 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$