

Name: _____

ID#: _____

Test # 1

MAT 127 Spring 2005

Directions: There are 5 questions. You have until 10 PM (90 minutes). For credit, you must show all your work, using the backs of the pages if necessary. You may not use a calculator.

1. ____/20 2. ____/20 3. ____/20 4. ____/20 5. ____/20

Total Score. ____/100

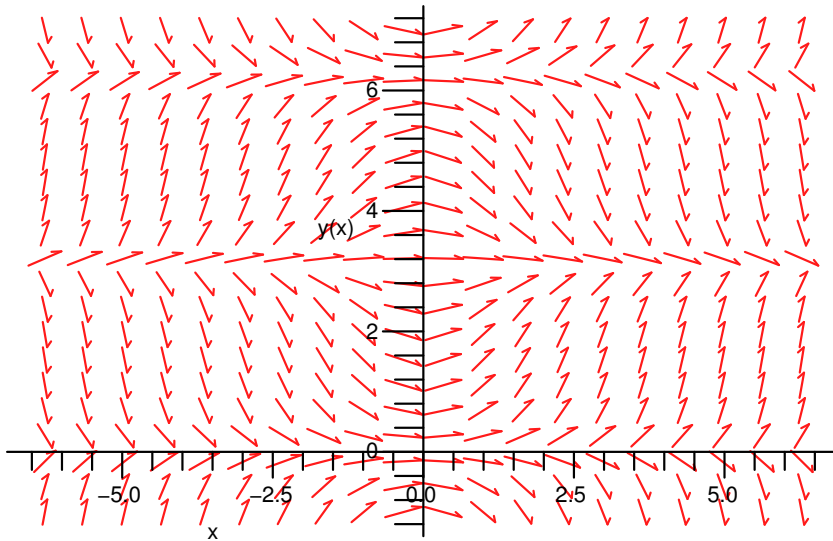
1. A function $y(t)$ satisfies the differential equation

$$\frac{dy}{dt} = y^4 - 5y^3 + 6y^2.$$

- (a) What are the constant solutions of the equation?
- (b) For what values of y is y increasing?
- (c) For what values of y is y decreasing?

2. A direction field for the differential equation $y' = x \sin y$ is shown.

- (a) Sketch the solutions that satisfy the given initial conditions (i) $y(0) = 1$, (ii) $y(0) = 4$, and (iii) $y(0) = \pi$. Label your graphs clearly.



- (b) Find all equilibrium solutions.

3. Use Euler's method with step size $1/2$ to estimate $y(2)$ for the solution to the initial value problem

$$y' = y - 2x \quad y(1) = 0.$$

4. Solve the following initial value problems. (Hint: the differential equations are separable.)

(a) $\frac{dx}{dt} = 1 + x + t + tx, x(0) = 0$

(b) $\frac{dy}{dt} = 2te^y, y(1) = 0$

5. Assume a population of well fed rabbits grows at a rate proportional to its size. Initially there are 100 rabbits and after 10 months there are 500 rabbits.

(a) Find an expression for the number of rabbits after t months.

(b) When will there be 5000 rabbits?