MATH 126 First Midterm

Wednesday, February 24, 2010

Name: ________________________ **ID:** ___________________ **Rec:** ____

Question:	1	2	3	4	5	6	Total
Points:	40	40	40	40	40	50	250
Score:							

There are 6 problems in this exam, printed on 6 pages (not including this cover sheet). Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate **clearly** what is where if you expect someone to look at it. **Books, calculators, extra papers, and discussions with friends are not permitted.** You may confer with a monkey, if you brought one with you. Leave all answers in exact form (that is, do *not* approximate π, square roots, and so on.)

You must give a correct justification of your answers to receive credit.

You have 90 minutes to complete this exam.

Name: __________________________ **Id:** __________________________

1. Let

$$f(x) = \frac{1}{2}x^2 + \frac{1}{2}, \qquad 1 \le x \le 5.$$

(In what follows you are not asked to compute the exact value of a definite integral.)

$\boxed{\text{20 points}}$ (a) Use the left endpoint rule and four intervals to estimate the area of the region under the graph of the function.

$\boxed{\text{10 points}}$ (b) Is the estimate you have obtained an overestimate or an underestimate?

$\boxed{\text{10 points}}$ (c) If you apply the right endpoint rule to the same function, interval and number of intervals, do you obtain an overestimate or an underestimate?

2. Consider the limit of the sum

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left(\frac{2}{n}i\right)^3. \tag{1}$$

20 points

(a) Find a function $f(x)$ and an interval $[a, b]$ such that the limit above is the area under the graph of $f(x)$ over the interval $[a, b]$.

20 points

(b) Use the following formula for the sum of the first n cubes

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

to compute the limit of the sum in (1) (do not compute the integral directly).

3.

$\boxed{\text{20 points}}$ (a) A function $f(x)$ has maximum value -2 and minimum value -5 on the interval $[-3, 3]$. Between what two values must

$$\int_{-2}^{2} f(x)\,dx$$

lie?

$\boxed{\text{20 points}}$ (b) Without computing the actual value, use the properties of definite integrals to estimate from above and from below the integral

$$\int_{-1}^{2} 1 + 2e^{x^2}\,dx.$$

You must give a correct justification of your answer to receive credit.

4. Evaluate the definite integrals in parts (a) and (b). Determine the general indefinite integrals in parts (c) and (d).

10 points (a) $\displaystyle\int_{-1}^{1} \frac{2}{t^2+1}\,dt$

10 points (b) $\displaystyle\int_{0}^{3} |2x-1|\,dx$

10 points (c) $\displaystyle\int \sec y \tan y\,dy$

10 points (d) $\displaystyle\int (e^x + \frac{1}{x} - \frac{3}{x^3})\,dx$

5. A particle is moving along a line [1]. Its velocity $v(t)$ at time 0 is equal to: $v(0) = 0$. Its acceleration is known and equal to:

$$a(t) = 3t - 6, \qquad 0 \le t \le 2.$$

20 points

(a) Find the velocity $v(t)$ in the given interval of time.

20 points

(b) The position $s(t)$ of the particle at time 0 is equal to: $s(0) = 0$. Find its position at time $t = 2$.

[1] It does not matter for your answer, but in case you wonder: distance is measured in meters and time is measured in seconds.

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6. Use the fundamental theorem of calculus (FTC) to answer the following questions.

15 points

(a) Find the derivative $g'(x)$ of the function $g(x) = \int_x^1 \sin(t^2 + 1)\,dt$.

15 points

(b) Find the derivative $h'(x)$ of the function $h(x) = \int_{\sin x}^2 e^{2t}\,dt$.

20 points

(c) Use the FTC to define the function $f(x)$ with these properties: $f'(x) = 2^x e^{\sin x}$ and $f(1) = 1$.