MY NAME IS:

Problem	1	2	3	4	Total
Score					

MAT 126 Calculus B Midterm 2, Solutions

April 5, 2000

SHOW ALL YOUR WORK ON THESE PAGES! TOTAL SCORE = 100

1. (30 points) Evaluate each of the following definite integrals.

(a) $\int_0^1 \sin(\pi x)\, dx$

Solution: Let $u = \pi x$. Then $du = \pi\, dx$. When $x = 0, u = 0$, and when $x = 1$, $u = \pi$. Thus

$$\int_0^1 \sin(\pi x)\, dx = \int_0^\pi \frac{\sin(u)}{\pi}\, du = -\frac{\cos(u)}{\pi}\bigg|_0^\pi = \frac{-\cos(\pi)}{\pi} - \frac{-\cos(0)}{\pi} = \frac{1}{\pi} + \frac{1}{\pi} = \frac{2}{\pi}$$

(b) $\int_0^1 x\sqrt{1-x^2}\, dx$

Solution: Let $u = 1 - x^2$. Then $du = -2x\, dx$. When $x = 0$, $u = 1$ and when $x = 1$, $u = 0$. Thus,

$$\int_0^1 x\sqrt{1-x^2}\, dx = -\frac{1}{2}\int_1^0 \sqrt{u}\, du = -\frac{1}{2}\cdot\frac{2}{3}x^{\frac{3}{2}}\bigg|_1^0 = -\frac{1}{3}\Big(0-1\Big) = \frac{1}{3}.$$

(c) $\int_{-1}^1 xe^x\, dx$

Solution: We integrate by parts. Take $u = x$ and $dv = e^x\, dx$. Then $du = dx$ and $v = e^x$. So we have

$$\int_{-1}^1 xe^x\, dx = xe^x\bigg|_0^1 - \int_{-1}^1 e^x\, dx = \Big(xe^x - e^x\Big)\bigg|_{-1}^1 = (e^1 - e^1) - (-e^{-1} - e^{-1}) = 2e^{-1} = \frac{2}{e}.$$

2. (30 points) Find the following antiderivatives.

(a) $\int x^2 \sin x \; dx$

Solution: We integrate by parts twice. First, $u = x^2$ and $dv = \sin x \, dx$, giving $du = 2x \, dx$ and $v = -\cos x$. So, we have

$$\int x^2 \sin x \; dx = -x^2 \cos x + 2 \int x \cos x \, dx.$$

To do the second integral, we take $u = x$ and $dv = \cos x \, dx$. Then $du = dx$ and $v = \sin x$. So, we have

$$\int x^2 \sin x \; dx = -x^2 \cos x + 2 \left(x \sin x - \int \sin x \, dx \right) = -x^2 \cos x + 2x \sin x + 2 \cos x + C.$$

(b) $\int \frac{x-1}{x^2+1} \; dx$

Solution:

$$\int \frac{x-1}{x^2+1} \; dx = \int \frac{x}{x^2+1} \; dx - \int \frac{1}{x^2+1} \; dx$$

To do the first integral, let $u = x^2 + 1$ so $du = 2x \, dx$. The second is immediate. We have

$$\int \frac{x-1}{x^2+1} \; dx = \frac{1}{2} \ln \left| x^2 + 1 \right| + \arctan(x) + C$$

(c) $\int e^{-x} \sin(2x) \; dx$

Solution: We integrate by parts twice, and solve.

First, take $u = \sin(2x)$ and $dv = e^{-x} \, dx$, giving $du = 2\cos(2x)$ and $v = -e^{-x}$. This gives

$$\int e^{-x} \sin(2x) \; dx = -e^{-x} \sin(2x) + 2 \int e^{-x} \cos(2x) \, dx$$

Now take $u = \cos(2x)$ and $dv = e^{-x} \, dx$, giving $du = -2 \sin(2x)$ and $v = -e^{-x}$.

$$\int e^{-x} \sin(2x) \; dx = -e^{-x} \sin(2x) + 2 \left(-e^{-x} \cos(2x) - 2 \int e^{-x} \sin(2x) \; dx \right).$$

That is, we've shown

$$\int e^{-x} \sin(2x) \; dx = -e^{-x} \sin(2x) - 2e^{-x} \cos(2x) - 4 \int e^{-x} \sin(2x) \; dx,$$

or, solving for the integral, we have

$$5 \int e^{-x} \sin(2x) \; dx = -e^{-x} \sin(2x) - 2e^{-x} \cos(2x) + C.$$

That is,

$$\int e^{-x} \sin(2x) \; dx = -\frac{1}{5} e^{-x} \sin(2x) - \frac{2}{5} e^{-x} \cos(2x) + C$$

3. (20 points) The function f is given by the table of values below.

x	0	0.5	1	1.5	2	2.5	3
$f(x)$	1	0.96	0.84	0.66	0.45	0.24	0.05

Approximate $\int_0^3 f(x)\,dx$ by using

(a) the left sum with 3 subintervals

Solution: Using 3 intervals, we have $\Delta x = 1$, so our approximation is

$$L_3 = \Big(f(0) + f(1) + f(2)\Big) = \Big(1 + 0.84 + 0.45\Big) = 2.29.$$

(b) the trapezoid rule with 3 subintervals

Solution:

$$T_3 = \frac{1}{2}\Big(f(0) + 2f(1) + 2f(2) + f(3)\Big) = \frac{1}{2}\Big(1 + 1.68 + 0.9 + 0.1\Big) = \frac{3.68}{2} = 1.84.$$

(c) Simpson's rule with 6 subintervals.

Solution: With 6 intervals, $\Delta x = 0.5$, so

$$\begin{aligned} S_6 &= \frac{0.5}{3}\Big(f(0) + 4f(0.5) + 2f(1) + 4f(1.5) + 2f(2) + 4f(2.5) + f(3)\Big) \\ &= \frac{1}{6}\Big(1 + 3.84 + 1.68 + 2.64 + 0.9 + 0.96 + 0.05\Big) \\ &= \frac{11.07}{6} = 0.9225. \end{aligned}$$

4. (20 points)

Evaluate the following indefinite integrals.

(a) $\displaystyle\int \frac{2\,dx}{(2+x)(3-x)}$

Solution: We use partial fractions to separate the integrand. Let

$$\frac{2}{(2+x)(3-x)} = \frac{A}{2+x} + \frac{B}{3-x}$$

so

$$2 = A(3-x) + B(2+x).$$

Thus

$$\begin{aligned} 2 &= 3A + 2B \\ 0 &= -A + B \end{aligned}$$

So $A = B$ and so $5A = 2$. That is, $A = B = 2/5$. So we have

$$\int \frac{2\,dx}{(2+x)(3-x)} = \int \frac{2/5}{2+x} + \frac{2/5}{3-x}\,dx = \frac{2}{5}\ln|2+x| - \frac{2}{5}\ln|3-x| + C.$$

(We get the negative sign on the second term by letting $u = 3 - x$ so $du = -dx$.)

(b) $\int \sin^3(x)\cos^4(x)\ dx$

Solution: Use the identity $\sin^2 x = 1 - \cos^2 x$ to rewrite the integral as follows

$$\int \sin^3(x)\cos^4(x)\ dx = \int \sin(x)(1-\cos^2(x))\cos^4(x)\ dx = -\int (1-u^2)u^4\ du$$

where we took $u = \cos(x)$ so $du = -\sin(x)\,dx$. Now

$$-\int (1-u^2)u^4\ du = \int u^6 - u^4\ dx = \frac{u^7}{7} - \frac{u^5}{5} + C$$

Thus, we have

$$\int \sin^3(x)\cos^4(x)\ dx = \frac{\cos^7(x)}{7} - \frac{\cos^5(x)}{5} + C.$$