MAT126, Paper Homework 5

1. After being kidnapped on a trip to Canada, little Jimmy was able to see the speedometer of the kidnapper's car, and carefully noted down the speeds every tenth of an hour (six minutes). (Since they are in Canada,the speeds are in km/hr). Jimmy was able to communicate these speeds to his friend Juan via text message, and Juan wants to figure out how far away Jimmy is from where they grabbed him.

time t	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1
speed $v(t)$	30	60	81	93	94	80	70	40	52	20	0

Using the midpoint rule, estimate the total distance (in km) that Jimmy was taken in the one hour, that is, estimate the integral $\int_0^1 v(t)\,dt$. (Use only the data provided– don't average to guess at speeds between the given times.)

Under the assumption that $-10 \le v''(t) \le 10$, also determine the maximum error in your estimate.

2. Approximate $\int_0^1 \ln(x)\cos(x)\,dx$ using Simpson's rule with 4 intervals, and give an estimate on the error of your approximation.

You can get around the problem of $\ln(0)\cos(0)$ not being defined as follows. First integrate by parts, and then it is reasonable to take $\sin(0)/0 = 1$ since $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$.

Similarly, you can use L'Hospital's rule to determine that $\lim_{x\to 0^+} \ln(x)\sin(x) = 0$.

So that you don't kill yourself taking derivatives, I'll tell you that the 4th derivative of $\sin(x)/x$ is $((x^4 - 12x^2 + 24)\sin(x) + (4x^3 - 24)\cos(x))/x^5$, which is between $-1/5$ and $1/5$ for any value of x.