MAT 126 - MIDTERM I - SPRING '08 SOL'NS

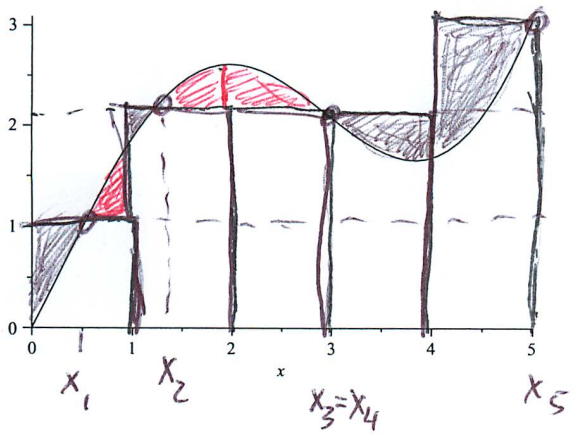


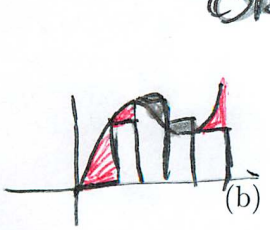
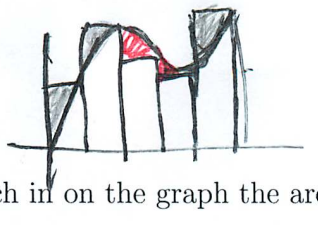
Figure 2: The graph of $f(x)$.

1. (a) (15 points) The figure shows the graph of a function $f(x)$ defined for $0 \le x \le 5$. Estimate the area under this graph as the sum of five rectangles based on $[0, 1]$, $[1, 2]$, $[2, 3]$, $[3, 4]$, $[4, 5]$, where the height of each rectangle is the value of f at the left-hand endpoint of its base.

THERE ARE MANY REASONABLE ANSWERS TO THIS PROBLEM. HERE'S ONE

$$\underset{\text{WIDTH}}{1}(1+2+2+2+3) = 10$$

OR YOU CAN USE RIGHT OR LEFT END POINTS BUT THOSE USE FRACTIONS, ITS OK IF YOU DID


OR


(b) (5 points) Sketch in on the graph the areas representing the *difference*:

[estimate from part (a)] *minus* [area under graph].

Label them "+" or "-" according as they are places where the estimate is too large, or places where the estimate is too small. Make sure your work is clear enough for the graders to understand what you mean.

+ IS IN BLACK
− IS IN RED

FROM THE SKETCH, IT LOOKS LIKE THE RED & BLACK CANCEL OUT FOR $0 < x < 4$, BUT ~~TOO~~ ABOUT 1 FOR $4 < x < 5$ SO (9) IS A BETTER EST.

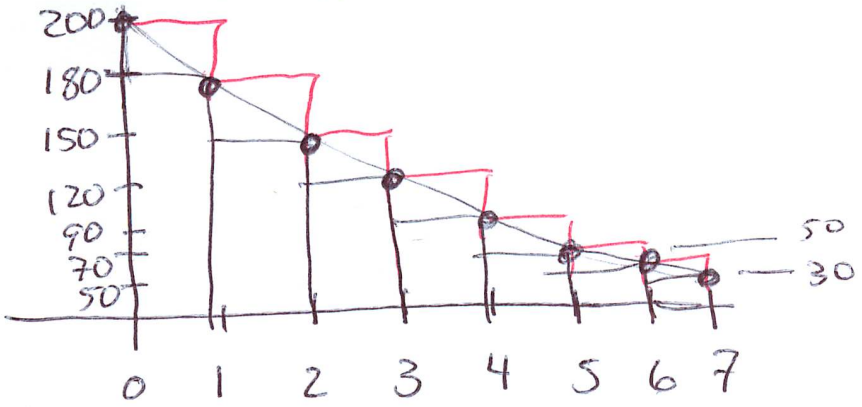
MAT126 - MID1 - SPR'08 SOLNS

2. (20 points) An Air Chance passenger plane touches down with ground speed 200 feet per second (136.4 mph). The pilot immediately puts the engines into reverse thrust to slow the plane down. The speed at t seconds after touchdown is recorded in the following table.

time in seconds	0	1	2	3	4	5	6	7
speed in feet/sec	200	180	150	120	90	70	50	30

After 7 seconds the plane is traveling at 30 feet/sec and can proceed at that speed to the terminal.

How far did the plane travel while it was decelerating to taxiing speed? Use the speeds in the table to get an *upper bound* and a *lower bound* for this distance.



GRAPH NOT NECESSARY BUT HELPFUL.

LOWER BOUND:

$1(180 + 150 + 120 + 90 + 70 + 50 + 30)$

$= 690$ FT

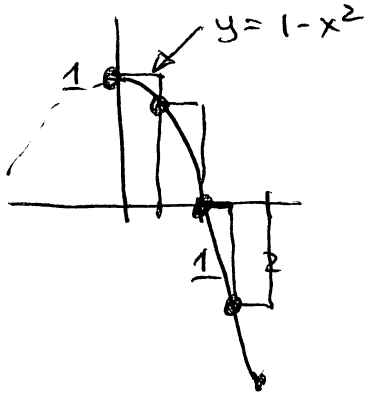
UPPER BOUND:

$1(200 + 180 + 150 + 120 + 90 + 70 + 50)$

$= 860$ FT

MAT126 - MID1 - SPR'08 - SOLNS

3. (a) (15 points) Estimate $\int_0^2 (1-x^2)\,dx$ by a Riemann sum with four equal subintervals. Use the left-hand endpoints as your x_i^*.



$$\frac{1}{2}\left(f(0) + f\left(\tfrac{1}{2}\right) + f(1) + f\left(\tfrac{3}{2}\right)\right)$$

$$= \frac{1}{2}\left(1 + \tfrac{3}{4} + 0 + \left(-\tfrac{1}{4}\right)\right)$$

$$= \frac{1}{2}\left(\tfrac{3}{2}\right) = \frac{3}{4}.$$

(b) (5 points) Is this sum an overestimate or an underestimate? Explain carefully.

THIS IS AN OVER-ESTIMATE BECAUSE $f(x)$ IS DECREASING ON $[0,2]$. THUS, EVERY RECTANGLE LIES ABOVE THE GRAPH.

(OR: SINCE $\int_0^2 (1-x^2)\,dx = x - \frac{x^3}{3}\Big|_0^2$

$$= \left(2 - \frac{8}{3}\right) - 0 = -\frac{2}{3}$$

AND $\frac{3}{4} > -\frac{2}{3}$, IT IS AN OVER-ESTIMATE)

MAT1726 - MIDI - SPRING 2018 SOLNS

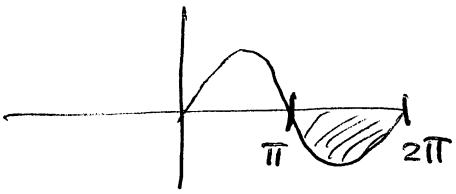
4. Calculate (show all work; NO CREDIT for unjustified answers):

(a) (8 points) $\int \sqrt[3]{x^2}\, dx$ $= \int x^{2/3} dx = \frac{3}{5} x^{5/3} + C$

(b) (8 points) $\int_{\pi}^{2\pi} \sin x\, dx$

$= -\cos x \Big|_{\pi}^{2\pi} = -\cos(2\pi) + \cos(\pi)$

$= -1 - 1 = -2$



(c) (4 points) $\int_0^t \frac{1}{(x^2+1)}\, dx$

$= \arctan(x) \Big|_0^t =$

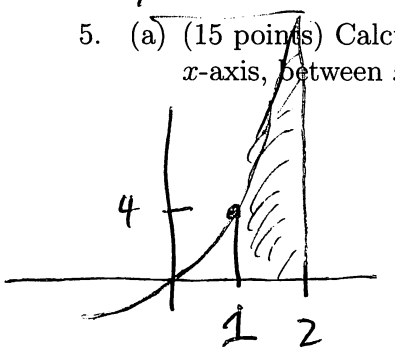
$= \arctan(t) - \arctan(0)$

$= \arctan(t)$.

MAT126 - MID1 - SPRÓG SOLS.

10

5. (a) (15 points) Calculate the area under the graph of $f(x) = x^2 + 3x$ and above the x-axis, between $x = 1$ and $x = 2$.



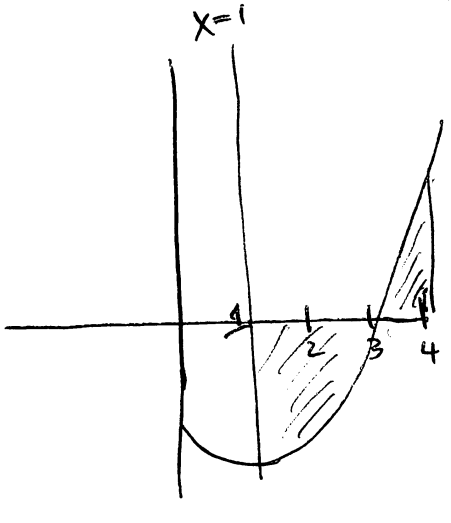
$$\int_1^2 (x^2+3x)\,dx = \frac{1}{3}x^3 + \frac{3}{2}x^2 \Big|_1^2$$

$$= \left(\frac{8}{3} + \frac{12}{2}\right) - \left(\frac{1}{3} + \frac{3}{2}\right)$$

$$= \frac{7}{3} + \frac{9}{2}$$

$$= \frac{14}{6} + \frac{27}{6} = \frac{41}{6}$$

(a) (5 points) Sketch the total area enclosed by the graph of $f(x) = x^2 - x - 6$, the x-axis, and the vertical lines $x = 1$ and $x = 4$; and calculate it exactly. Note that $x^2 - x - 6 = (x+2)(x-3)$, so the function changes sign at 3.



WANT AREA NOT INTEGRAL
(THAT IS, AREA FROM 1 TO 3 NEED TO CHANGE SIGN)

$$\int_1^3 (x^2 - x + 6)\,dx = \frac{x^3}{3} - \frac{x^2}{2} - 6x \Big|_1^3$$

$$= \left(\frac{27}{3} - \frac{9}{2} - 18\right) - \left(\frac{1}{3} - \frac{1}{2} - 6\right)$$

$$= \frac{26}{3} - 4 - 12 = -\frac{22}{3}$$

$$\int_3^4 (x^2 - x - 6)\,dx$$

$$= \frac{x^3}{3} - \frac{x^2}{2} - 6x \Big|_3^4 = \left(\frac{64}{3} - \frac{16}{2} - 24\right) - \left(\frac{27}{3} - \frac{9}{2} - 18\right)$$

$$= \frac{17}{6}$$

AREA IS $\frac{22}{3} + \frac{17}{6} = \frac{61}{6}$

END OF EXAMINATION