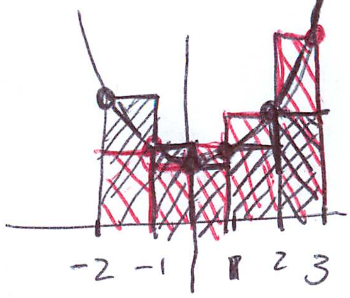
Stony Brook University
Dept. of Mathematics

MAT 126 Calculus B
Sept. 30, 2015

Please show all of your work.

1) Approximate the area between the graph of $y = x^2 + 6$ and the x-axis, on the interval $[-2,3]$ using $n = 5$:

$n=5$ MEANS $\Delta x = \frac{3+2}{5} = 1$.



a) Left Endpoint Rectangles

LEFT ENDPOINT MEANS WE HAVE TO EVALUATE AT $-2, -1, 0, 1, 2$

WE HAVE

$$1(f(-2) + f(-1) + f(0) + f(1) + f(2))$$
$$= 10 + 7 + 6 + 7 + 10$$
$$= 40$$

Answer (5 points)

40

b) Right Endpoint Rectangles, SO WE EVALUATE AT $-1, 0, 1, 2, 3$

$$1(f(-1) + f(0) + f(1) + f(2) + f(3))$$
$$= 7 + 6 + 7 + 10 + 15$$
$$= 45$$

Answer (5 points)

45

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2) $\int_{-2}^{3}\left(x^2+6\right)dx =$ $\frac{1}{3}x^3 + 6x \Big|_{-2}^{3}$

$= \left(\frac{27}{3} + 18\right) - \left(\frac{-8}{3} - 12\right)$

$= 9 + 18 + \frac{8}{3} + 12$

$= \frac{125}{3}$

Answer (10 points)

$\frac{125}{3}$ OR $41\frac{2}{3}$

3) If $f(x)=\int\left(e^x+\pi x\right)dx$ and $f(0)=2$, find $f(x)$.

$f(x) = e^x + \frac{\pi}{2}x^2 + C$

SINCE $f(0) = 2 = e^0 + C = 1 + C$, $C = 1$

Answer (10 points)

$f(x) = e^x + \frac{\pi}{2}x^2 + 1$

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4) Find $\frac{d}{dx}\int_{\cos x}^{\sin x} e^{3t}dt$

BY THE FUNDAMENTAL THEOREM OF CALCULUS,

WE GET $e^{3\sin x} \cdot \cos x - e^{3\cos x} \cdot (-\sin x)$

ALTERNATIVELY, THIS IS

$$\frac{d}{dx}\left[\frac{1}{3}e^{3t}\Big|_{\cos x}^{\sin x}\right] = \frac{d}{dx}\left(\frac{1}{3}e^{3\sin x} - \frac{1}{3}e^{3\cos x}\right)$$

$$= \cos x \cdot e^{3\sin x} + \sin x \cdot e^{3\cos x},$$

SAME....

Answer (10 points)

$$\boxed{\cos x \cdot e^{3\sin x} + \sin x \cdot e^{3\cos x}}$$

5) $\int x\sqrt[3]{5-2x^2}dx = \int x(5-2x^2)^{1/3}dx$

LET $w = 5-2x^2$

$dw = -4x\,dx$, SO $-\frac{1}{4}dw = x\,dx$.

$$= -\frac{1}{4}\int w^{1/3}dw$$

$$= -\frac{1}{4}\cdot\frac{3}{4}w^{4/3} + C$$

$$\boxed{= -\frac{3}{16}(5-2x^2)^{4/3} + C}$$

Please show all of your work.

6) $\int x \sin 5x \, dx =$

INTEGRATE BY PARTS WITH

SO
$$u = x \qquad dv = \sin 5x\,dx$$
$$du = dx \qquad V = \int \sin 5x\,dx = -\frac{1}{5}\cos 5x$$

THEN

$$\int x \sin(5x)dx = -\frac{1}{5}x\cos(5x) + \frac{1}{5}\int \cos(5x)dx + C$$

$$= -\frac{1}{5}x\cos(5x) + \frac{1}{25}\sin(5x) + C$$

CHECK: $\frac{d}{dx}\left(-\frac{1}{5}x\cos(5x) + \frac{1}{25}\sin(5x) + C\right)$

PRODUCT

$$= \left(-\frac{1}{5}\cos(5x) + x\sin 5x\right) + \frac{5}{25}\cos(5x)$$

CANCEL

$$= x \sin 5x$$

☺

Answer (10 points)

$$\boxed{-\frac{1}{5}x\cos(5x) + \frac{1}{25}\sin(5x) + C}$$

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7) $\int \frac{1+2x}{\sqrt{1-x^2}}dx = \int \frac{1}{\sqrt{1-x^2}}dx + \int \frac{2x}{\sqrt{1-x^2}}dx$

LET $u = 1-x^2$ so $du = -2x\,dx$

$= \arcsin(x) - \int (u)^{-1/2} du$

$= \arcsin(x) - 2u^{1/2} = \arcsin x - 2\sqrt{1-x^2} + c$

CHECK:

$\frac{d}{dx}\left(\arcsin x - 2\sqrt{1-x^2}\right) = \frac{1}{\sqrt{1-x^2}} - 2\left(\frac{1}{2}\right)(1-x^2)^{-1/2}\cdot(-2x)$

$= \frac{1}{\sqrt{1-x^2}} + \frac{2x}{\sqrt{1-x^2}} = \frac{1+2x}{\sqrt{1-x^2}}$ YAY!.

Answer (10 points)

$\arcsin x - 2\sqrt{1-x^2} + c$

8) $\int_1^{e^2} x \ln x \, dx =$

BY PARTS, WITH $u = \ln x \quad dv = x\,dx$; $du = \frac{1}{x}dx \quad v = \frac{1}{2}x^2$

$= \frac{1}{2}x^2 \ln x \Big|_1^{e^2} - \int_1^{e^2} \left(\frac{1}{2}x^2\right)\left(\frac{1}{x}dx\right)$

$= \left(\frac{e^4}{2}\underbrace{\ln e^2}_{=2} - \frac{1}{2}\underbrace{\ln 1}_{=0}\right) - \frac{1}{2}\int_1^{e^2} x\,dx$

$= e^4 - \left(\frac{1}{4}x^2\Big|_1^{e^2}\right) = e^4 - \left(\frac{e^4}{4} - \frac{1}{4}\right) = \frac{3}{4}e^4 + \frac{1}{4}$

Answer (10 points)

$\frac{1}{4} + \frac{3}{4}e^4$

Please show all of your work.

9) $\int \frac{\sec^2 x - \sec^2 x \sin^2 x}{\cos^2 x} dx =$ $\int \frac{\sec^2 x\,(1-\sin^2 x)}{\cos^2 x}\,dx$

SINCE
$\sin^2 x + \cos^2 x = 1$
$\cos^2 x = 1 - \sin^2 x$

$= \int \frac{\sec^2 x\,(\cos^2 x)}{\cos^2 x}\,dx$

$= \int \sec^2 x\,dx$

$= \tan x + C$

CHECK: $\frac{d}{dx} \tan x = \sec^2 x$.

Answer (10 points)

$\boxed{\tan x + C}$.

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10) $\int e^x \sin(2x)\,dx =$

BY PARTS (TWICE): LET $u = e^x$ $dv = \sin 2x$
$du = e^x dx$ $v = -\frac{1}{2}\cos 2x$

$\int e^x \sin(2x)dx = -\frac{1}{2}e^x\cos(2x) + \frac{1}{2}\int e^x\cos 2x\,dx$

LET $u = e^x$ $dv = \cos 2x\,dx$
$du = e^x dx$ $v = \frac{1}{2}\sin 2x$

$\int e^x\sin(2x)dx = -\frac{1}{2}e^x\cos(2x) + \frac{1}{2}\left(\frac{1}{2}e^x\sin(2x) - \frac{1}{2}\int e^x\sin 2x\,dx\right)$

$\int e^x\sin 2x\,dx = -\frac{1}{2}e^x\cos(2x) + \frac{1}{4}e^x\sin 2x - \frac{1}{4}\int e^x\sin 2x\,dx$
$+\frac{1}{4}\int e^x\sin 2x\,dx$

SO $+\frac{1}{4}\int e^x\sin 2x dx$

$\frac{5}{4}\int e^x\sin 2x\,dx = -\frac{1}{2}e^x\cos(2x) + \frac{1}{4}e^x\sin(2x) + C$

$\therefore \int e^x\sin 2x\,dx = \frac{4}{5}\left(-\frac{1}{2}e^x\cos 2x + \frac{1}{4}e^x\sin 2x\right) + C$

$= \frac{1}{5}e^x\sin(2x) - \frac{2}{5}e^x\cos(2x) + C$

Answer (10 points)

$$\boxed{\frac{e^x\sin 2x}{5} - \frac{2e^x\cos(2x)}{5} + C}$$