

$$\textcircled{1} \int (3x-1)^5 dx \quad \begin{array}{l} \text{let } u = 3x-1 \\ du = 3 dx \\ \frac{1}{3} du = dx \end{array}$$

$$= \frac{1}{3} \int u^5 du$$

$$= \frac{1}{3} \left( \frac{u^6}{6} \right) + C = \frac{u^6}{18} + C = \frac{(3x-1)^6}{18} + C$$


---

$$\textcircled{2} \int x(2-x^2)^3 dx \quad \begin{array}{l} \text{let } u = 2-x^2 \\ du = -2x dx \\ -\frac{1}{2} du = x dx \end{array}$$

$$= \frac{1}{2} \int u^3 du$$

$$= \frac{1}{2} \left( \frac{u^4}{4} \right) + C = \frac{-u^4}{8} + C = -\frac{(2-x^2)^4}{8} + C$$


---

$$\textcircled{3} \int \sin 3x dx \quad \begin{array}{l} \text{let } u = 3x \\ du = 3 dx \\ \frac{1}{3} du = dx \end{array}$$

$$= \frac{1}{3} \int \sin u du$$

$$= -\frac{1}{3} \cos u + C = -\frac{1}{3} \cos(3x) + C$$


---

$$\textcircled{4} \int \sqrt{3x-1} dx \quad \begin{array}{l} \text{let } u = 3x-1 \\ du = 3 dx \\ \frac{1}{3} du = dx \end{array}$$

$$= \frac{1}{3} \int u^{\frac{1}{2}} du$$

$$= \frac{1}{3} \frac{u^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{9} u^{\frac{3}{2}} + C = \frac{2}{9} (3x-1)^{\frac{3}{2}} + C$$

$$\begin{aligned}
 (5) \quad & \int x \sqrt{7x^2+12} \, dx \quad \text{let } u=7x^2+12 \\
 & \quad \quad \quad du=14x \, dx \\
 & \quad \quad \quad \frac{1}{14} du = x \, dx \\
 & = \frac{1}{14} \int u^{\frac{1}{2}} \, du \\
 & = \frac{1}{14} \frac{u^{\frac{3}{2}}}{\frac{3}{2}} = \frac{1}{21} u^{\frac{3}{2}} + C = \frac{1}{21} (7x^2+12)^{\frac{3}{2}} + C
 \end{aligned}$$


---

$$\begin{aligned}
 (6) \quad & \int \frac{x}{(4x^2+1)^3} \, dx \quad \text{let } u=4x^2+1 \\
 & \quad \quad \quad du=8x \, dx \\
 & \quad \quad \quad \frac{1}{8} du = x \, dx \\
 & = \frac{1}{8} \int \frac{du}{u^3} = \frac{1}{8} \int u^{-3} \, du \\
 & = \frac{1}{8} \frac{u^{-2}}{-2} = -\frac{1}{16} \cdot \frac{1}{u^2} = -\frac{1}{16} \cdot \frac{1}{(4x^2+1)^2} \, dx
 \end{aligned}$$


---

$$\begin{aligned}
 (7) \quad & \int \frac{\sin(\frac{5}{x})}{x^2} \, dx \quad \text{let } u = \frac{5}{x} \\
 & \quad \quad \quad du = -\frac{5}{x^2} \, dx \\
 & \quad \quad \quad -\frac{1}{5} du = \frac{dx}{x^2} \\
 & = -\frac{1}{5} \int \sin u \, du \\
 & = \frac{1}{5} \cos u + C = \frac{1}{5} \cos\left(\frac{5}{x}\right) + C
 \end{aligned}$$

$$\textcircled{8} \int \frac{\sec^2(\sqrt{x})}{\sqrt{x}} dx \quad \text{let } u = \sqrt{x}$$

$$du = \frac{1}{2\sqrt{x}} dx$$

$$2du = \frac{dx}{\sqrt{x}}$$

$$= 2 \int \sec^2 u du$$

$$= 2 \tan u + C = 2 \tan \sqrt{x} + C$$

---

$$\textcircled{9} \int \sin(\sin \theta) \cos \theta d\theta \quad \text{let } u = \sin \theta$$

$$du = \cos \theta d\theta$$

$$= \int \sin u du = -\cos u + C = -\cos(\sin \theta) + C$$

---

$$\textcircled{10} \int x \sqrt{x-3} dx \quad \text{let } u = x-3$$

$$u+3 = x$$

$$du = dx$$

$$= \int (u+3) \sqrt{u} du$$

$$= \int u^{\frac{3}{2}} + 3u^{\frac{1}{2}} du = \frac{u^{\frac{5}{2}}}{\frac{5}{2}} + \frac{3u^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= \frac{2}{5} u^{\frac{5}{2}} + 2u^{\frac{3}{2}} + C$$

$$= \frac{2}{5} (x-3)^{\frac{5}{2}} + 2(x-3)^{\frac{3}{2}} + C$$

$$(11) \int x e^{-x} dx \quad u = x \quad dv = e^{-x} dx$$

$$du = dx \quad v = -e^{-x}$$

$$\int x e^{-x} dx = -x e^{-x} + \int e^{-x} dx$$

$$= -x e^{-x} - e^{-x} + C$$

$$(12) \int \ln(2x+3) dx \quad u = \ln(2x+3) \quad dv = dx$$

$$du = \frac{2}{2x+3} dx \quad v = x$$

$$\int \ln(2x+3) dx = x \ln(2x+3) - \int \frac{2x}{2x+3} dx$$

$$= x \ln(2x+3) - \int \left( 1 - \frac{3}{2x+3} \right) dx$$

$$\begin{aligned} \frac{2x}{2x+3} &= \frac{2x+3-3}{2x+3} \\ &= \frac{2x+3}{2x+3} - \frac{3}{2x+3} \end{aligned}$$

$$= x \ln(2x+3) - \left( x - 3 \frac{\ln(2x+3)}{2} \right) + C$$

$$(13) \int x \ln x dx \quad u = \ln x \quad dv = x dx$$

$$du = \frac{1}{x} dx \quad v = \frac{x^2}{2}$$

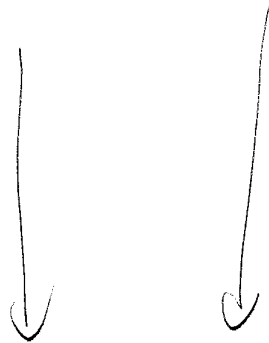
$$\int x \ln x dx = \frac{x^2}{2} \ln x - \frac{1}{2} \int x dx$$

$$= \frac{x^2}{2} \ln x - \frac{1}{2} \left( \frac{x^2}{2} \right) + C$$

$$\textcircled{14} \quad \int \sin^{-1} x \, dx \quad u = \sin^{-1} x \quad dv = dx$$

$$du = \frac{1}{\sqrt{1-x^2}} dx \quad v = x$$

$$\int \sin^{-1} x \, dx = x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} dx$$



$$\text{let } u = 1-x^2$$

$$du = -2x dx$$

$$\frac{1}{2} du = x dx$$

$$= -\frac{1}{2} \int u^{-\frac{1}{2}} du = -\frac{u^{\frac{1}{2}}}{\frac{1}{2}} = -u^{\frac{1}{2}}$$

$$= -\sqrt{1-x^2}$$

$$\int \sin^{-1} x \, dx = x \sin^{-1} x + \sqrt{1-x^2} + C$$

$$\textcircled{15} \quad \int x^2 e^{-x} dx \quad u = x^2 \quad dv = e^{-x} dx$$

$$du = 2x dx \quad v = -e^{-x}$$

$$\int x^2 e^{-x} dx = -x^2 e^{-x} + 2 \int x e^{-x} dx$$

$$u = x \quad dv = e^{-x} dx$$

$$du = dx \quad v = -e^{-x}$$

$$= -x^2 e^{-x} + 2(-x e^{-x} + \int e^{-x} dx)$$

$$= -x^2 e^{-x} - 2x e^{-x} + 2 \int e^{-x} dx$$

$$= -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + C$$

$$(16) \int e^x \sin x dx \quad u = e^x \quad dv = \sin x dx$$

$$du = e^x dx \quad v = -\cos x$$

$$\int e^x \sin x dx = -e^x \cos x + \int e^x \cos x dx$$

$$u = e^x \quad dv = \cos x dx$$

$$du = e^x dx \quad v = \sin x$$

$$\int e^x \sin x dx = -e^x \cos x + e^x \sin x - \int e^x \sin x dx$$

add this integral to both sides

$$2 \int e^x \sin x dx = -e^x \cos x + e^x \sin x + C$$

$$\int e^x \sin x dx = \frac{-e^x \cos x + e^x \sin x}{2} + C$$

$$(17) \int x \tan^{-1} x dx \quad u = \tan^{-1} x \quad dv = x dx$$

$$du = \frac{1}{1+x^2} dx \quad v = \frac{x^2}{2}$$

$$\int x \tan^{-1} x dx = \frac{x^2}{2} \tan^{-1} x + \frac{1}{2} \int \frac{x^2}{1+x^2} dx$$

$$= \frac{x^2}{2} \tan^{-1} x + \frac{1}{2} \int \left( \frac{x^2}{1+x^2} \right) dx$$

USE  $\frac{x^2}{1+x^2} = \frac{1+x^2-1}{1+x^2} = \frac{1+x^2}{1+x^2} - \frac{1}{1+x^2} = 1 - \frac{1}{1+x^2}$

$$= \frac{x^2}{2} \tan^{-1} x + \frac{1}{2} \left( x - \tan^{-1} x \right) + C$$

$$\textcircled{19} \int x^2 \ln x \, dx \quad \begin{array}{l} u = \ln x \\ du = \frac{1}{x} dx \end{array} \quad \begin{array}{l} dv = x^2 dx \\ v = \frac{x^3}{3} \end{array}$$

$$\begin{aligned} \int x^2 \ln x \, dx &= \frac{x^3}{3} \ln x - \frac{1}{3} \int x^2 dx \\ &= \frac{x^3}{3} \ln x - \frac{1}{3} \cdot \frac{x^3}{3} + C \end{aligned}$$

$$\textcircled{20} \int x \cos(3x) \, dx \quad \begin{array}{l} u = x \\ du = dx \end{array} \quad \begin{array}{l} dv = \cos(3x) dx \\ v = \frac{\sin(3x)}{3} \end{array}$$

$$\begin{aligned} \int x \cos(3x) \, dx &= \frac{x \sin(3x)}{3} - \frac{1}{3} \int \sin(3x) \, dx \\ &= \frac{x \sin(3x)}{3} - \frac{1}{3} \left( \frac{-\cos(3x)}{3} \right) + C \\ &= \frac{x \sin(3x)}{3} + \frac{1}{9} \cos(3x) + C \end{aligned}$$