MATH 126   FINAL

Monday, May 17 2010

Name: _____   **ID:** _____   **Rec:** _____

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	40	30	40	30	30	30	40	40	40	30	350
Score:											

There are 10 problems in this exam, printed on 10 pages (not including this cover sheet). Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate **clearly** what is where if you expect someone to look at it. **Books, calculators, extra papers, and discussions with friends are not permitted.** Leave all answers in exact form (that is, do *not* approximate π, square roots, and so on.)

Sorry, there is no secret message here to give you extra credit. However, perhaps you can rub the tail of the lemur below for extra good luck.

You must give a correct justification of all answers to receive credit.

You have $2\frac{1}{2}$ hours to complete this exam.

20 pts   1. (a) Express the following limit of Riemann sums as a definite integral.
              (do not compute the integral)

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{x_i^2 + e^{x_i}}\, \Delta x, \qquad \text{where } x_i = 2 + i\Delta x, \quad \Delta x = \frac{1}{n}$$

20 pts   (b) Express the following integral as a limit of Riemann sums.
              (do not compute the integral)

$$\int_1^3 \sqrt{1 + x^3}\, dx$$

30 pts   2. Define a function $f(x) = \int_0^{x^2} \sqrt{t + \sqrt{t}} \ dt.$

Find the value of $f'(1)$.

3. Determine whether each integral is convergent or divergent and evaluate those that are convergent (if any).

20 pts   (a) $\displaystyle\int_{-\infty}^{0} \frac{1}{(-1-x)^{\frac{1}{3}}}\, dx$
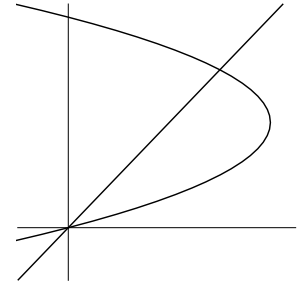
20 pts   (b) $\displaystyle\int_{0}^{1} \frac{\ln x}{x}\, dx$

30 pts 4. Find the area of the region bounded by the two curves

$$x = 3y - y^2 \qquad \text{and} \qquad y = x.$$

(a) Write an integral which represents this area.

(b) Evaluate the integral in (a).

30 pts   5. Compute the following integral. If the integral diverges, write "divergent".
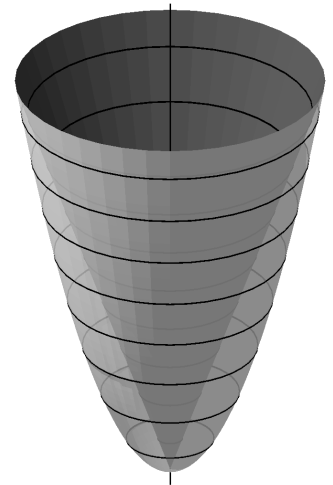
$$\int_{-1}^{0} \frac{x}{(x-1)(x-2)}\, dx.$$

30 pts 6. Find the volume of the solid obtained by rotating the region between the two curves

$$y = 2x \qquad \text{and} \qquad y = x^2$$

about the y-axis.

(a) Write an integral which represents the volume.

(b) Evaluate the integral in (a).

20 pts   7. (a) Compute the definite integral

$$\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$$

20 pts   (b) Compute the indefinite integral $\int (\sec 2t)(\tan 2t)\, dt$

20 pts 8. (a) Compute the following indefinite integral $\displaystyle\int p^6 \ln p \ dp$

20 pts   (b) Compute the following definite integral $\displaystyle\int_0^\pi t \sin 3t \ dt.$

20 pts   9. (a) Write an equation in Cartesian (x-y) coordinates for the curve with polar equation $r = 2\sin\theta$.   Your answer should not contain trigonometric functions.

20 pts   (b) Find the $(a + ib)$-form of the complex number

$$\left[\frac{1 + i\sqrt{3}}{2}\right]^{20}$$

This question covers material (complex numbers) that we did not cover in Fall 22.

30 pts 10. Find the center of mass of a flat plate with uniform density that occupies the region bounded by the two curves $y = 2$ and $y = 3 - x^2$.

This question requires material we did not cover. Our exam will have different applications.