MAT 126 Final Exam

December 13, 2018

Name: ______________________ **ID:** __________________ **Rec:** ____

Question:	1	2	3	4	5	6*	7*	8*	9*	10*	Total
Points:	40	20	10	15	20	20	20	20	20	20	185
Score:											

There are 10 problems on 9 pages in this exam. Make sure that you have them all.

Note that you should **cross out one of problems 6–10**, and not do it.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. If you read these instructions, write the phrase i is unreal at the bottom of this page and I will give you 2 points of extra credit on the exam. **Books, calculators, extra papers, and discussions with friends are not permitted. No electronic devices may be used AT ALL.** Since we studied complex numbers, conferring with imaginary friends is allowed, provided you do so quietly. Conferring with enemies (real or imaginary) is not recommended (nor is it permitted).

Points will be taken off for writing mathematically false statements, even if the rest of the problem is correct.

Leave all answers in exact form (that is, do *not* approximate π, square roots, and so on.)

You might find it helpful to remember the following:

$$\begin{aligned} \sin^2\theta + \cos^2\theta &= 1 & \tan^2\theta + 1 &= \sec^2\theta \\ \sin^2\theta &= \tfrac{1}{2}\big(1-\cos(2\theta)\big) & \cos^2\theta &= \tfrac{1}{2}\big(1+\cos(2\theta)\big) \\ \sin(2\theta) &= 2\cos\theta\sin\theta & \cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ e^{i\theta} &= \cos\theta + i\sin\theta & e^{i\pi}+1 &= 0 \end{aligned}$$

Or maybe not. Who can tell?

You have 2 hours and 30 minutes to complete this exam.

Name: ______________________ **Id:** ______________________

1. Evaluate each of the following integrals. If the integral diverges, write "diverges". You must justify your answers for full credit.

$\boxed{\text{10 pts}}$ (a) $\displaystyle\int \frac{4x-2}{x^2-x+2}\,dx$

$\boxed{\text{10 pts}}$ (b) $\displaystyle\int_1^\infty \frac{1}{x^3}\,dx$

$\boxed{\text{10 pts}}$ (c) $\displaystyle\int xe^{4x}\,dx$

$\boxed{\text{10 pts}}$ (d) $\displaystyle\int_0^{2\pi} |\sin(x)|\,dx$

20 pts 2. Find the area of the region bounded by the graphs of the two curves $y = x^2 - 2x$ and $y = x + 4$.

10 pts

3. Express the integral $\int_{-2}^{4} x^2 + 2x\,dx$ as a limit of a Riemann sum (with n rectangles). Your final answer should not include symbols like Δx or x_i.

15 pts

4. Find the average value of $f(x) = \arctan 3x$ for $0 \leq x \leq \frac{1}{3}$.

$\boxed{\text{20 pts}}$ 5. Recall that $\displaystyle\int_1^5 \frac{dx}{x} = \ln 5$. Use Simpson's rule with $n = 4$ to write a fraction (or sum of fractions) approximating $\ln 5$.

If you've forgotten Simpson's rule, you can use the Trapezoid rule, but you will lose 5 points for doing so. If you've forgotten that, try using a right-endpoint Riemann sum to maybe get 10 pts.

Name: ____________________ **Id:** ____________________

Do **any four** of the five questions 6-10. Cross out the one you don't want graded.

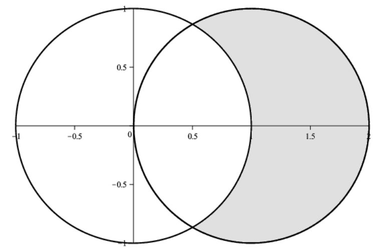
20 pts 6. Compute the following integral. If the integral diverges, write "divergent".

$$\int_{2}^{4/\sqrt{2}} \frac{dx}{x^2\sqrt{16-x^2}}$$

Do **any four** of the five questions 6-10. Cross out the one you don't want graded.

20 pts

7. Find the area of the region that lies inside the circle of radius one centered at $(1, 0)$, but outside the circle of radius one centered at the origin.

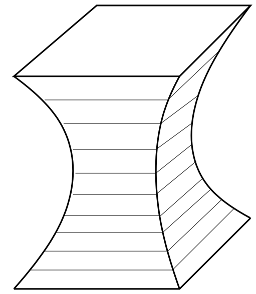


You can do this either in polar coordinates (where the two curves are given by $r = 2\cos\theta$ and $r = 1$) or in rectangular coordinates (where the curves are given by $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$). In the rectangular case, you need to cut up the area appropriately.

Do **any four** of the five questions 6-10. Cross out the one you don't want graded.

20 pts

8. A pillar that is π feet tall is made so that every horizontal cross section at height h is a square of side length
$$s(h) = 3 - \sin(h).$$



(a) Write an integral which represents the volume of the pillar.

(b) Evaluate the integral to find the volume of the pillar.

Name: ____________________ **Id:** ____________________

Do **any four** of the five questions 6-10. Cross out the one you don't want graded.

20 pts 9. Compute the following integral. If the integral diverges, write "divergent".

$$\int_1^2 \frac{x^2+2x+3}{x^2(3-x)}\,dx$$

Name: ________________________ **Id:** ________________________

Do **any four** of the five questions 6-10. Cross out the one you don't want graded.

20 pts 10. After a certain treatment, the probability of the symptoms of a specific illness recurring t years after the treatment is given by a log-logistic distribution[1] of the form

$$f(t) = \begin{cases} \dfrac{18t}{(t^2+9)^2} & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}.$$

After the treatment, what is the probability that the illness will not recur for at least 5 years?

[1] This is also known as a Fisk distribution when used in economics. This distribution depends on two parameters: a scale parameter α and a shape parameter β; for this treatment, $\alpha = 3$ and $\beta = 2$.