Math 126   Second Midterm

November 2, 2022

Name: _____   **ID:** _____   **Rec:** _____

Question:	1	2	3	4	5	Total
Points:	30	30	15	15	30	120
Score:						

There are 5 problems on 4 pages (plus this cover sheet) in this exam. Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of the pages, but indicate what is where if you expect someone to look at it.

Books, calculators, extra papers, and discussions with friends are not permitted. No electronic devices may be used AT ALL. [Insert facetious remark about impossible way to cheat here.]

Points will be taken off for writing mathematically false statements, even if the rest of the problem is correct.

Use non-erasable pen (not red) if you want to be able to contest the grading of any problems. Questions with erasures will not be regraded.

Leave all answers in exact form (that is, **do *not* approximate π, square roots, and so on**.) You might find it helpful to remember the following (or not):

$$\sin^2 \theta + \cos^2 \theta = 1 \qquad \tan^2 \theta + 1 = \sec^2 \theta$$
$$\sin^2 \theta = \tfrac{1}{2}\big(1 - \cos(2\theta)\big) \qquad \cos^2 \theta = \tfrac{1}{2}\big(1 + \cos(2\theta)\big)$$

You have 90 minutes to complete this exam.

1. Compute each of the following definite integrals. If the integral does not converge, write "Diverges".   In all cases you must justify your answer to receive full credit.

10 pts   (a) $\int_2^\infty \frac{dx}{x^4}$

10 pts   (b) $\int_0^{\pi/2} \sin(z)\cos^2(z)\,dz$

10 pts   (c) $\int_0^3 \frac{dw}{(w-1)^5}$

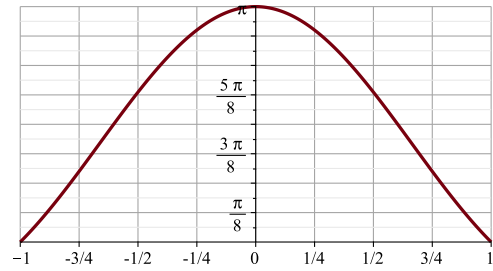**30 pts** 2. At right is shown the graph of the function

$$f(x) = \begin{cases} \frac{\sin \pi x}{x} & x \neq 0 \\ \pi & x = 0 \end{cases}.$$

(a) Use the trapezoid method with 4 intervals to approximate the value of $\int_{-1}^{1} f(x)\, dx$.

(b) Estimate the error[1] in your answer in the previous part. You might find it helpful to know that $|f'(x)| < 13/3$ or that $-21/2 < -\pi^3/3 \leq f''(x) \leq 2\pi < 13/2$ for $|x| \leq 1$.

(c) How many intervals are required to ensure that the error is less than $1/100$ when using the trapezoid rule to estimate this integral?

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[1]The max. error in the trapezoid approximation to $\int_a^b f(x)dx$ using n intervals is $\frac{K(b-a)^3}{12n^2}$, where $K \geq |f''(x)|$ for all $a \leq x \leq b$.   Also, $\sqrt{2} \approx 1.414$, $\sqrt{3} \approx 1.732$, $\sqrt{7} \approx 2.646$, $\sqrt{13} \approx 3.606$ and $\sqrt{21} \approx 4.583$.

15 pts   3. Evaluate the indefinite integral $\int \dfrac{10 \, dx}{(x+3)(x^2+1)}$.

Use K for an unknown constant. You must show all steps to receive credit.

15 pts   4. Find the average value of $f(x) = x^2$ for $-3 \le x \le 2$.

5. Let R be the region with $x \geq 0$ and bounded by the graphs of $y = x^2 + 2$ and $y = x + 4$.

| 15 pts |

    (a) Calculate the area of the region R.

| 15 pts |

    (b) Calculate the volume of the solid obtained by rotating R around the x-axis.