MAT 126 First Midterm

September 29, 2022

Name: _____ **ID:** _____ **Rec:** ____

Question:	1	2	3	4	5	Total
Points:	20	10	15	20	15	80
Score:						

There are 5 problems on 5 pages (plus this cover sheet) in this exam. Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of the pages, but indicate what is where if you expect someone to look at it.

Books, calculators, extra papers, and discussions with friends are not permitted. No electronic devices may be used AT ALL. Feel free to communicate via the spirit realm with Bernhard Riemann for assistance with any of these problems. However, you almost certainly will need to be fluent in German as spoken about 200 years ago for it to do you any good. (If you have done this, please make an appropriate acknowledgment to avoid plagiarism issues.)

Points will be taken off for writing mathematically false statements, even if the rest of the problem is correct.

Use non-erasable pen (not red) if you want to be able to contest the grading of any problems. Questions with erasures will not be regraded.

Leave all answers in exact form (that is, **do *not* approximate π,square roots, and so on**.)

You have 90 minutes to complete this exam.

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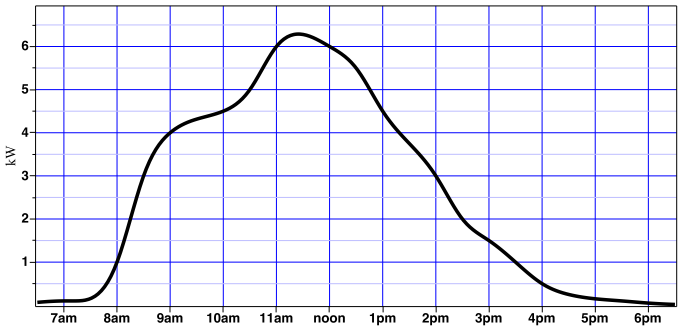
20 pts

1. At right is shown a graph of the amount of electricity generated (in kilowatts) by a home solar array on a recent day, as a function of time. Let $E(t)$ denote this function. Then the total power produced (in kilowatt-hours) between times $t = a$ and $t = b$ is given by the integral $\int_a^b E(t)\,dt$.



(a) Use a Riemann sum with three intervals of equal width evaluated at **the right endpoint**, to calculate the total power generated between 9am and 3pm.

(b) Use a Riemann sum with three intervals of equal width evaluated at **the midpoint**, to calculate the total power generated between 9am and 3pm.

2. Evaluate each of the indefinite integrals below. An answer given with no justification will receive little or no credit. Use C for an unknown constant, where necessary.

5 pts (a) $\int e^{1-5t}\, dt$

5 pts (b) $\int \theta \sin 4\theta\, d\theta$

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3. Evaluate each of the definite integrals below. An answer given with no justification will receive little or no credit. Use C for an unknown constant, where necessary.

5 pts (a) $\displaystyle\int_0^{\sqrt{3}} \frac{3x+1}{1+x^2}\,dx$

5 pts (b) $\displaystyle\int_{-1}^{2} \left|w^3\right|\,dw$

5 pts (c) $\displaystyle\int_1^3 \ln(1+z)\,dz$

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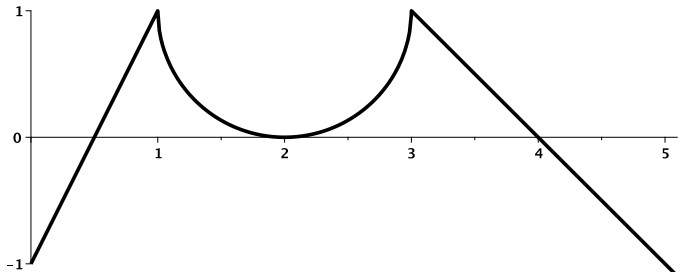
4. Let $g(t)$ be the function given by

$$g(t) = \begin{cases} 2t - 1 & t \leq 1 \\ 1 - \sqrt{1 - (t-2)^2} & 1 < t \leq 3 \\ 4 - t & t > 3 \end{cases}$$



with its graph shown at right. Let

$$F(x) = \int_0^x g(t)\, dt\ .$$

For full credit, give at least a little justification for each of your answers to the questions below.

5 pts (a) For what values of x between 0 and 5 is $F(x) \leq 0$?
Write as intervals. If there are none, write "None".

5 pts (b) What is $F(1)$?
If $F(1)$ is not defined, write "DNE".

5 pts (c) What is $F'(1)$?
If $F'(1)$ does not exist, write "DNE".

5 pts (d) What is $F''(4)$?
If $F''(4)$ does not exist, write "DNE".

5. The limit

$$\lim_{n\to\infty} \frac{4}{n} \sum_{k=1}^{n} \ln\left(1 + \frac{8k}{n}\right)$$

corresponds to a definite integral.

10 pts (a) State a definite integral that the limit corresponds to. To get full credit, you must give some explanation of how this integral and the limit are related. Do not calculate the integral.

5 pts (b) Is the sum $\frac{4}{10} \sum_{k=1}^{10} \ln\left(1 + \frac{8k}{10}\right)$ larger or smaller than the integral above? Why? (You should not need to calculate either the integral or the sum in order to answer this question).