Stony Brook

STATE UNIVERSITY OF NEW YORK

MAT 126 Midterm 2 – Fall 2017

Problem	1	2	3	4	5	6	7	8	9	10	Total
Score											

Last Name: SOLUTIONS First Name:________________

Recitation #: _______ (See below for your recitation number) Student ID # ______________________

Directions: Answer all questions in the space provided. You may use the blank backs of pages for scrap. No other paper is permitted. Show ALL relevant work. Calculators are not to be used. Circle your final answers.

Schedule

R01	F	10:00am-10:53am	Library	W4530	Mohamed El Alami
R02	Tu	1:00pm- 1:53pm	Mathematics	P131	Thomas Rico
R03	Th	4:00pm- 4:53pm	Earth & Space	69	David Bishop
R04	W	5:30pm- 6:23pm	Earth & Space	69	Tobias Shin
R05	W	4:00pm- 4:53pm	Physics	P116	Tobias Shin
R20	M	12:00pm-12:53pm	Harriman Hall	112	Selin Taşkent
R21	Th	10:00am-10:53am	Library	E4310	Ying Hong Tham
R23	Tu	4:00pm- 4:53pm	Library	N4072	Mohamed El Alami
R30	M	5:30pm- 6:23pm	Library	N4006	TanyaLisa Agha
R31	M	4:00pm- 4:53pm	Library	N4006	TanyaLisa Agha
R32	Th	2:30pm- 3:23pm	Library	E4310	Thomas Rico
R33	Th	7:00pm- 7:53pm	Library	E4310	Selin Taşkent

F'17 SOL'NS

Directions: Show all work in the space provided. If you need more space use the blank backs of the exam sheets. Be sure that you don't have answers to any question in more than one place. Answers without the required work will receive no credit. Simplify your answers.

Part I: For the following find the indefinite or definite integrals. Leave all answers in simplest form. [10 points each]

Reference Formulas: You may not need all of these.

$$cos^2x = \frac{1}{2}(1 + cos2x) \qquad sin^2x = \frac{1}{2}(1 - cos2x)$$

$$sin^2x + cos^2x = 1 \qquad tan^2x = sec^2x - 1$$

$$a^2\,cos^2\theta = a^2 - a^2sin^2\theta$$

$$a^2\,tan^2\theta = a^2sec^2\theta - a^2 \qquad a^2\,sec^2\theta = a^2tan^2\theta + a^2$$

1. $\displaystyle\int_0^{\pi/3} 2e^{\cos x}\sin x\,dx$

LET $u = \cos x$, $du = -\sin x\,dx$

$x = 0 \Rightarrow u = 1$

$x = \pi/3 \Rightarrow u = \cos(\frac{\pi}{3}) = \frac{1}{2}$

$$= -\int_1^{1/2} 2e^u\,du$$

$$= \int_{1/2}^{1} 2e^u\,du = 2e^u\Big|_{1/2}^{1} = \boxed{2e - 2e^{1/2}}$$

$$= 2(e - \sqrt{e})$$

2. $\int x^2 \sin x \, dx$ INTEGRATE BY PARTS (TWICE) $\left[\int u\,dv = uv - \int v\,du\right]$

$\begin{bmatrix} u = x^2 & dv = \sin x\,dx \\ du = 2x\,dx & v = -\cos x \end{bmatrix}$

$= -x^2\cos x - \left(-2\int x\cos x\,dx\right)$ $\begin{bmatrix} u = x & dv = \cos x\,dx \\ du = dx & v = \sin x \end{bmatrix}$

$= -x^2\cos x + 2x\sin x - 2\int \sin x\,dx$

$= \boxed{-x^2\cos x + 2x\sin x + 2\cos x + C}$

3. $\int (1 - sin^2 x)\, dx$ USE $\sin^2 x = \frac{1}{2}(1-\cos 2x)$ $\left[\begin{array}{l} \text{OR } 1-\sin^2 x \\ = \cos^2 x, \\ = \frac{1}{2}(1+\cos 2x) \end{array}\right.$

$= \int 1 - \frac{1}{2} + \frac{1}{2}\cos 2x\,dx = \int\left(\frac{1}{2} + \frac{1}{2}\cos 2x\right)dx$

$= \boxed{\frac{1}{2}x + \frac{1}{4}\sin 2x + C}$

4. $\int \frac{x}{\csc(x^2)}\,dx$ LET $u = x^2$
$du = 2x\,dx$

$= \frac{1}{2}\int \frac{du}{\csc(u)}$ USE $\csc(u) = \frac{1}{\sin(u)}$

$= \frac{1}{2}\int \sin(u)\,du = -\frac{1}{2}\cos(u) + C$

$= \boxed{-\frac{1}{2}\cos(x^2) + C}$

$= \frac{-1}{2\sec(x^2)} + C$ IF YOU PREFER.

5. $\int \sec^3 x \tan x\, dx = \int (\sec^2 x)(\sec x \tan x\, dx)$

LET $w = \sec x$
$dw = \sec x \tan x\, dx$

$= \int w^2\,dw$

$= \frac{1}{3}w^3 + C$

$= \frac{1}{3}\sec^3 x + C$

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6. $\int \frac{x^2+1}{x^2-2x}\,dx =$

$= \int 1 + \frac{2x+1}{x(x-2)}\,dx$

$= x + \int \frac{-1/2}{x}\,dx + \int \frac{5/2}{x-2}\,dx$

$= \boxed{x - \frac{1}{2}\ln|x| + \frac{5}{2}\ln|x-2| + C}$

DIVIDE:

$$\begin{array}{r|l} & 1 + \frac{2x+1}{x^2-2x} \\ x^2-2x & x^2+1 \\ & x^2-2x \\ \hline & 2x+1 \end{array}$$

PARTIAL FRACTIONS:

$\frac{2x+1}{x(x-2)} = \frac{A}{x} + \frac{B}{x-2}$

$2x+1 = A(x-2) + Bx$

$x=0 \Rightarrow 1 = -2A + 0 \Rightarrow A = -1/2$

$x=2 \Rightarrow 5 = 0 + 2B \Rightarrow B = 5/2$

SO $\frac{2x+1}{x(x-2)} = \frac{-1/2}{x} + \frac{5/2}{x-2}$

7. $\int \cot x\; dx$ [hint: Write $\cot x$ another way]

$= \int \frac{\cos x}{\sin x}\,dx$ $\quad w = \sin x$, $dw = \cos x\,dx$

$= \int \frac{1}{w}\,dw$

$= \ln|w| + C$

$= \boxed{\ln|\sin x| + C}$

SOL'NS F'17

Part II: Answer the following. Leave all answers in simplest form. [10 points each]

8. Evaluate the improper integral below. If the integral diverges explain why, otherwise give the value of the integral.

$$\int_0^{\infty} \frac{x}{1+x^2}\,dx$$

BY COMPARISON WITH $\int_1^{\infty} \frac{1}{x}\,dx$ THIS $\boxed{\text{DIVERGES}}$ (SINCE $\frac{x}{1+x^2} > \frac{x}{x^2} = \frac{1}{x}$ AND $\int_1^{\infty} \frac{1}{x}\,dx$ DIVERGES)

OR

LET $u = 1+x^2$ $\quad x=0 \Rightarrow u=1$

$du = 2x\,dx$ $\quad x=\infty \Rightarrow u=\infty$

SO

$$\int_0^{\infty} \frac{x}{1+x^2}\,dx = \frac{1}{2}\int_1^{\infty} \frac{du}{u} = \lim_{M\to\infty} \ln|u|\Big|_1^M = \lim_{M\to\infty} \ln M - \ln 1$$

$$= +\infty$$

$\boxed{\text{DIVERGES}}$

9. Evaluate the improper integral below. If the integral diverges explain why, otherwise give the value of the integral.

$$\int_0^1 \frac{2dx}{\sqrt{1-x^2}}$$

IMPROPER AT $x=1$ (SINCE $\frac{2}{\sqrt{1-1}}$ NOT DEFINED)

$$= \lim_{b\to 1^-} \int_0^b \frac{2}{\sqrt{1-x^2}}\,dx$$

$x = \sin\theta$ $\quad x=b \Rightarrow \theta = \arcsin b$

$dx = \cos\theta\,d\theta$ $\quad x=1 \Rightarrow \theta = 0$

SO $\sqrt{1-x^2} = \sqrt{1-\sin^2\theta} = \sqrt{\cos^2\theta} = \cos\theta$

$$= \lim_{b\to 1^-} \int_0^{\arcsin b} \frac{2\cos\theta\,d\theta}{\cos\theta}$$

(OH CRAP, I FORGOT THAT $\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$!)

$$= \lim_{b\to 1^-} 2\theta\Big|_0^{\arcsin b} = \lim_{b\to 1} 2\arcsin b - 0$$

YOU DON'T NEED TO DO ALL THIS...

$$= 2\arcsin(1) = 2\cdot\frac{\pi}{2} = \boxed{\pi}$$

(SOL'NS F'17)

10. Evaluate the integral $\int \frac{dx}{x^2\sqrt{4-x^2}}$ dx using trig substitution. See the reference formulas on page 2 of the exam.

LET $x = 2\sin\theta$ so $\sqrt{4-x^2} = \sqrt{4-4\sin^2\theta}$

$dx = 2\cos\theta\, d\theta$ $\quad = \sqrt{4\cos^2\theta}$

$= 2\cos\theta$

$$\int \frac{dx}{x^2\sqrt{4-x^2}} = \int \frac{2\cos\theta\, d\theta}{(2\sin\theta)^2(2\cos\theta)}$$

$$= 2\int \frac{d\theta}{4\sin^2\theta} = \frac{1}{2}\int \csc^2\theta\, d\theta$$

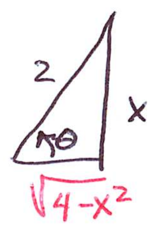
$$= -\frac{1}{2}\cot\theta \quad + C$$

NOW WE HAVE TO WRITE $\cot\theta$ IN TERMS OF X.

SINCE $x = 2\sin\theta$, $\sin\theta = x/2 = \frac{\text{OPP}}{\text{HYP}}$

SINCE $\cot\theta = \frac{1}{\tan\theta} = \frac{\text{ADJ}}{\text{OPP}}$

$\cot(\theta) = \frac{\sqrt{4-x^2}}{x}$



THUS

$$\boxed{\int \frac{dx}{x^2\sqrt{4-x^2}} = -\frac{1}{2}\frac{\sqrt{4-x^2}}{x} + C}$$