Math 126 Second Midterm

November 3, 2016

Name: ______________________ **ID:** __________________ **Rec:** ____

Question:	1	2	3	4	5	6	Total
Points:	30	30	30	30	30	30	180
Score:							

There are 6 problems in this exam. Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, calculators, extra papers, and discussions with friends are not permitted. No electronic devices may be used AT ALL.** If you have a Galaxy Note 7 phone, you may use it freely, but only *after* it has burst into flames.

Points will be taken off for writing mathematically false statements, even if the rest of the problem is correct.

Use non-erasable pen (not red) if you want to be able to contest the grading of any problems. Questions with erasures will not be regraded.

Leave all answers in exact form (that is, do *not* approximate π, square roots, and so on.) You might find it helpful to remember the following:

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta$$
$$\sin^2\theta = \tfrac{1}{2}\big(1 - \cos(2\theta)\big) \qquad \cos^2\theta = \tfrac{1}{2}\big(1 + \cos(2\theta)\big)$$

You have 90 minutes to complete this exam.

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$\boxed{\text{30 pts}}$ 1. Evaluate each of the integrals below. If the integral does not converge, write **Diverges**. An answer given with no justification will receive little or no credit.

(a) $\displaystyle\int_0^1 \frac{7x+1}{1+x^2}\,dx$

(b) $\displaystyle\int \sin^5(x)\cos^3(x)\,dx$

(c) $\displaystyle\int_1^\infty \frac{dx}{x^4}$

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30 pts 2. Recall that $\displaystyle\int_1^3 \frac{dx}{x} = \ln 3$.

(a) Use Simpson's rule with $n = 4$ (that is, 5 points) to approximate $\ln 3$.

(b) Estimate the error in your approximation above. The maximum error in Simpson's rule to approximate $\int_a^b f(x)\,dx$ is $\dfrac{M(b-a)^5}{180n^4}$, where M is the maximum of $|f^{(4)}(x)|$ for $a \le x \le b$.

(c) What n will ensure that Simpson's rule gives an answer correct to within ± 0.001?*

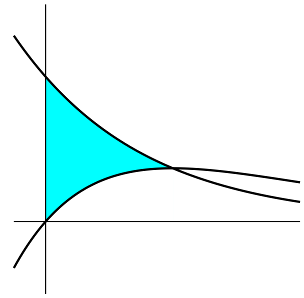
* $4^4 = 256, 5^4 = 625, 6^4 = 1296, 7^4 = 2401, 8^4 = 4096, 9^4 = 6561, 10^4 = 10000, 11^4 = 14641, 12^4 = 20736,$ $13^4 = 28561, 14^4 = 38416, 15^4 = 50625, 16^4 = 65535, 17^4 = 83521, 18^4 = 104976, 19^4 = 130321, 20^4 = 160000.$

30 pts

3. Consider the finite region in the first quadrant bounded by the curves

$$y = e^{-4x} \quad \text{and} \quad y = xe^{-4x},$$

corresponding to the shaded region in the figure at right.



(a) On the figure, label both curves and the three points at the "corners" of the region.

(b) Write an integral which represents the area of the region.

(c) Calculate the area of the region.

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30 pts

4. Zhulong has a candle store, and he sells tapered candles with a star-shaped cross-section, like the one shown at right.

He is setting up a process to manufacture these candles (he used to make them by hand), and so he needs a careful calculation of the volume of each one. The candles have sides that are tapered so that the diameter at height h is $4 - h/3$. The area of a regular five-pointed star inscribed in a circle of radius r is given by the formula

$$A(r) = \frac{10\tan\left(\frac{\pi}{10}\right)}{3 - \tan^2\left(\frac{\pi}{10}\right)}\, r^2 \;\approx\; \frac{9}{8} r^2.$$

What is the volume of a candle that is 3 inches tall?
(The approximation of $\frac{9}{8}$ is good within .001, so use that. You don't have to simplify the fractions.)

This question uses material that will not be covered on the Fall 2018 MAT126 exam, but it WILL be on the final exam.

30 pts 5. Compute the integral: $\displaystyle\int_2^3 \frac{6x^3 - 5x^2 - 4x + 1}{x^2(1+x)(1-x)}\,dx\ .$

Remember that an *exact* answer is wanted; do not approximate logarithms, exponentials, π, or square roots.

$\boxed{\text{30 pts}}$ 6. Compute the integral: $\displaystyle\int_0^{\frac{\sqrt{2}}{2}} \frac{4x^2\,dx}{\sqrt{1-x^2}}$.

Remember that an *exact* answer is wanted; do not approximate logarithms, exponentials, π, or square roots.