FALL '15

Stony Brook University
Dept. of Mathematics

SOLUTIONS

MAT 126 Calculus B
December 9, 2015

Part One – Minimum Competence

1) $\int_1^2 \left(3x^2 + 5x + 4\right)dx =$

$$x^3 + \frac{5x^2}{2} + 4x \Big|_1^2$$

$$= \left(8 + \frac{20}{2} + 8\right) - \left(1 + \frac{5}{2} + 4\right)$$

$$= 26 - 15/2 = \frac{37}{2} = 18.5$$

Answer (10 points)

$$\boxed{\frac{37}{2} = 18.5}$$

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2) $\int 2xe^x dx =$

INTEGRATE BY PARTS WITH

$u = 2x \quad dv = e^x dx$

$du = 2dx \quad v = e^x$

$= 2xe^x - 2\int e^x dx$

$= 2x e^x - 2e^x + C = (2x-2)e^x + C$

Answer (10 points)

$\boxed{(2x-2)e^x + C}$

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3) $\int_0^{\frac{\pi}{2}} \sin^2 x \cos x dx =$

LET $u = \sin x$, $du = \cos x dx$

IF $x = 0$, $u = \sin(0) = 0$

$x = \pi/2 \Rightarrow u = \sin(\pi/2) = 1$

$$\int_0^{\pi/2} \sin^2 x \cos x dx = \int_0^1 u^2 du = \frac{u^3}{3}\Big|_0^1 = 1/3 - 0$$

$$= 1/3$$

Answer (10 points)

$1/3$

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4) $\int \frac{x}{x^2+2}\,dx =$

LET $u = x^2+2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$\int \frac{x\,dx}{x^2+2} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln|x^2+2| + C$$

Answer (10 points)

$\boxed{1/2 \ln|x^2+2| + C}$

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5) $\displaystyle\int \frac{7x-5}{x^2-x-2}\,dx = \int \frac{7x-5}{(x-2)(x+1)}\,dx$

USING PARTIAL FRACTIONS:

$$\frac{7x-5}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1} \Rightarrow 7x-5 = A(x+1) + B(x-2)$$

LET $X=-1$: $-7-5 = 0 + B(-3) \Rightarrow B=4$

LET $X=2$: $14-5=9 = A(3) \Rightarrow A=3$

(ALTERNATIVELY, MULTIPLY IT OUT, GET
$7x-5 = Ax + A + Bx - 2B$. THUS $A+B=7$ ①
$A-2B=-5$ ②
① − ② $\Rightarrow 3B = +12 \Rightarrow B=4$, SO $A=3$.)

THUS $\displaystyle\int \frac{7x-5}{(x-2)(x+1)}\,dx = \int \frac{3}{x-2} + \frac{4}{x+1}\,dx = 3\ln|x-2| + 4\ln|x+1| + C$

Answer (10 points)

$$\boxed{3\ln|x-2| + 4\ln|x+1| + C}$$

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6) $\int_4^{\infty} e^{-2x}dx =$ $\lim_{M \to \infty} \int_4^{M} e^{-2x}\,dx$

$$= \lim_{m \to \infty} -\frac{1}{2}e^{-2x}\Big|_4^{m}$$

$$= \lim_{m \to \infty} \left(-\frac{1}{2}e^{-2M} + \frac{1}{2}e^{-8}\right)$$

$$= 0 + \frac{1}{2}e^{-8} = \frac{1}{2}e^{-8}$$

Answer (10 points)

$$\frac{1}{2}e^{-8} = \frac{1}{2e^{8}}$$

SOL'NS

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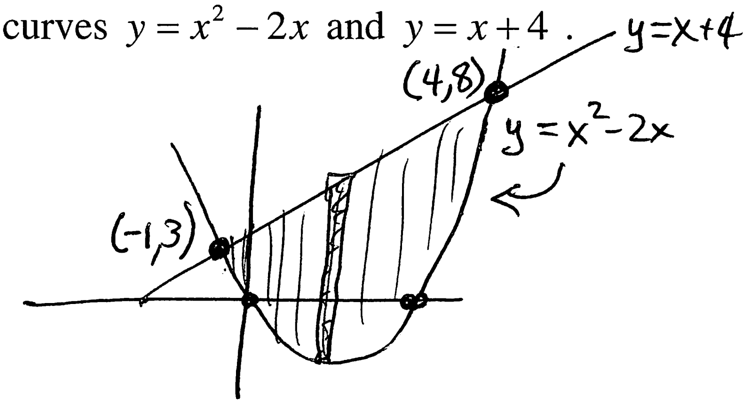
Part Two

7) The region R is formed by the curves $y = x^2 - 2x$ and $y = x + 4$.
Find the area of R.



WHERE DO THEY CROSS?

$x+4 = x^2-2x$

$0 = x^2-3x-4$

$0 = (x-4)(x+1)$

CROSS @ $x=4$ & $x=-1$

TYPICAL RECTANGLE AT x FOR $-1 < x < 4$

IS dx WIDE, $(x+4)-(x^2-2x)$ TALL.

AREA IS $\int_{-1}^{4} (x-4-x^2+2x)\,dx$

$= \frac{3}{2}x^2 - 4x - \frac{x^3}{3} \Big|_{-1}^{4}$

$= \left(\frac{3}{2}\cdot 16 - 16 - \frac{64}{3}\right) - \left(\frac{3}{2} + 4 + \frac{1}{3}\right)$

$= -\frac{40}{3} - \frac{35}{6} = -\frac{115}{6}$

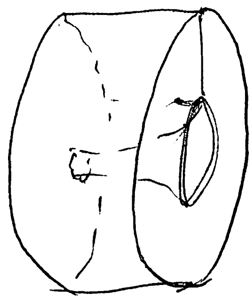
Answer (10 points)

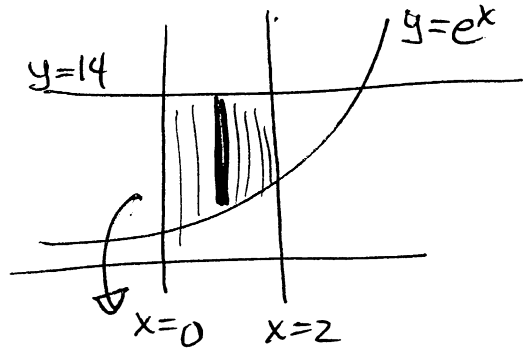
$-\frac{115}{6}$

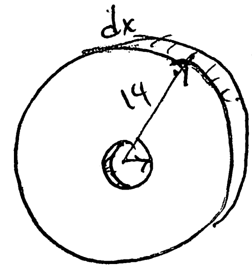
Sol'ns

8) The region R is formed by the curves $y = e^x$, $y = 14$, $x = 0$, and $x = 2$.

a) Find the volume of the solid that results when R is revolved around the x-axis using the Washer Method. ***Yes, you must evaluate this integral.***







TYPICAL WASHER HAS OUTER RADIUS 14, INNER RADIUS e^x.

AREA IS $\pi(14)^2 - \pi(e^x)^2$

$= \pi(4 \cdot 49 - e^{2x})$

VOLUME IS $\displaystyle\int_0^2 \pi(4 \cdot 49 - e^{2x})\,dx$

$= \pi\left(196x - \frac{1}{2}e^{2x}\right|_0^2$

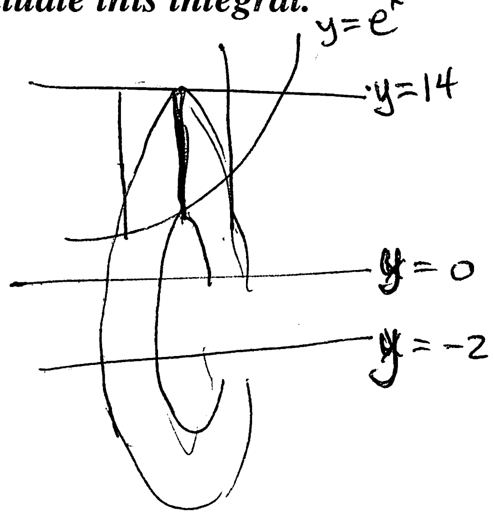
$= \pi\left(196 \cdot 2 - \frac{1}{2}e^4\right) - \pi\left(0 - \frac{1}{2}\right)$

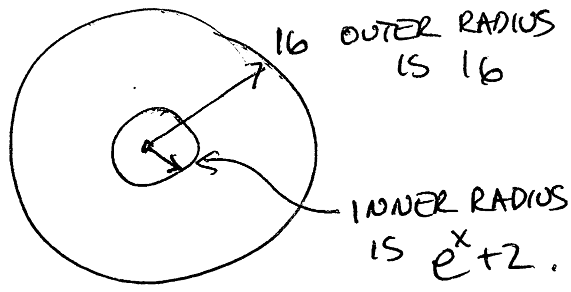
$= \pi\left(392 - \frac{1}{2}e^4 + \frac{1}{2}\right)$

Answer (10 points)

$\boxed{\frac{\pi}{2}(785 - e^4)}$

b) Find the volume of the solid that results when R is revolved around the line $y = -2$ using the Washer Method. ***Set up but do not evaluate this integral.***





AREA IS

$$\pi\left(16^2 - (e^x+2)^2\right)$$

VOLUME IS

$$\pi\int_0^2 \left(16^2 - (e^x+2)^2\right) dx$$

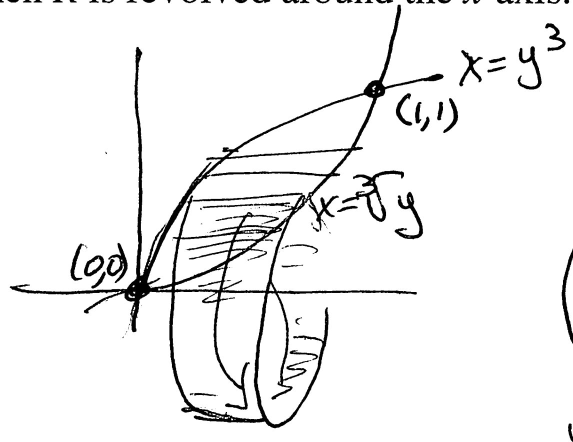
Answer (5 points)

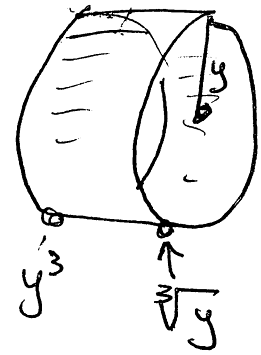
$$\boxed{\pi\int_0^2 \left(256 - (e^x+2)^2\right) dx}$$

SOL'NS

9) The region R is formed by the curves $x = y^3$ and $x = \sqrt[3]{y}$ in the first quadrant. Use the Shell Method to find the volume of the solid that results when R is revolved around the x-axis.

CROSS AT $(0,0)$ & $(1,1)$





TYPICAL SHELL AT HEIGHT y HAS RADIUS y (CIRCUMFERENCE $2\pi y$)

HEIGHT $(\sqrt[3]{y} - y^3)$

AREA IS $2\pi y(\sqrt[3]{y} - y^3)$.

VOLUME IS

$$\int_0^1 2\pi y(\sqrt[3]{y} - y^3)\,dy$$

$$= 2\pi\int_0^1 (y^{4/3} - y^4)\,dy = 2\pi\left(\frac{3}{7}y^{7/3} - \frac{1}{5}y^5\right)\Bigg|_0^1$$

$$= 2\pi\left(\frac{3}{7} - \frac{1}{5}\right) - 0$$

$$= 2\pi\left(\frac{15-7}{35}\right) = \frac{16\pi}{35}$$

Answer (10 points)

$16\pi/35$

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Soln's

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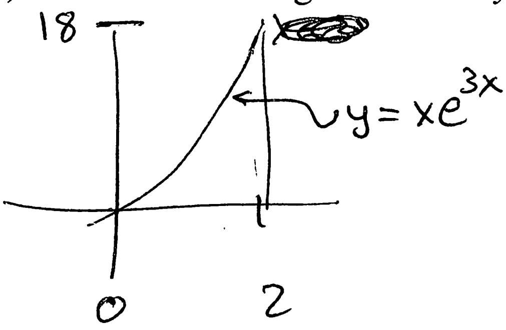
10) Find the average value of $y = xe^{3x}$ on the interval $[0,2]$.



$$\text{AVERAGE VALUE} = \frac{1}{2-0}\int_0^2 xe^{3x}\,dx$$

$$= \frac{1}{2}\int_0^2 xe^{3x}\,dx$$

BY PARTS
$u = x \quad dv = e^{3x}dx$
$du = dx \quad v = \frac{1}{3}e^{3x}$

$$= \frac{1}{2}\left(\frac{1}{3}xe^{3x}\Big|_0^2 - \frac{1}{3}\int_0^2 e^{3x}\,dx\right)$$

$$= \frac{1}{6}\left(xe^{3x} - \frac{1}{3}e^{3x}\Big|_0^2\right.$$

$$= \frac{1}{6}\left[\left(2e^6 - \frac{1}{3}e^6\right) - \left(0 - \frac{1}{3}\right)\right]$$

$$= \frac{1}{3}e^6 - \frac{1}{18}e^6 + \frac{1}{18} = \frac{1}{3}\left(\frac{5}{6}e^6 + \frac{1}{6}\right)$$

Answer (10 points)

$$\boxed{\frac{1}{3}\left(\frac{5}{6}e^6 + \frac{1}{6}\right) = \frac{5}{18}e^6 + \frac{1}{18}}$$

SOLNS

11) Find the length of the arc of the curve defined by $y = \frac{2}{3}x^{\frac{3}{2}} - 1$ on the interval $[0,1]$

ARC LENGTH IS $\int_0^1 \sqrt{1+[f'(x)]^2}\, dx$

$f(x) = \frac{2}{3}x^{3/2} - 1$

$f'(x) = x^{1/2}$ $\quad [f'(x)]^2 = x$

$AL = \int_0^1 \sqrt{1+x}\, dx$ $\quad u = 1+x$, $du = dx$ $\quad \cancel{x}=0 \Rightarrow u=1$, $x=1 \Rightarrow u=2$

$= \int_1^2 \sqrt{u}\, du$

$= \frac{2}{3}u^{3/2}\Big|_1^2 = \frac{2}{3}(\sqrt{8} - 1)$

Answer (15 points)

$$\boxed{\frac{2}{3}(\sqrt{8}-1) = \frac{4\sqrt{2}}{3} - \frac{2}{3}}$$

12) $\displaystyle\int \frac{11x^2-12x+5}{(x^2+1)(x-2)}\,dx =$ BY PARTIAL FRACTIONS.

$$\frac{11x^2-12x+5}{(x^2+1)(x-2)} = \frac{Ax+B}{x^2+1} + \frac{C}{x-2}$$

$$11x^2-12x+5 = (Ax+B)(x-2) + C(x^2+1)$$

$\underline{x=2} \Rightarrow \underbrace{44-24+5}_{25} = 0 + C(4+1) \Rightarrow 25 = 5C \Rightarrow \boxed{C=5}$

$\underline{x=0} \Rightarrow 5 = (0+B)(-2) + 5(1)$

$\Rightarrow 5 = -2B+5 \Rightarrow \boxed{B=0}$

$\underline{x=1} \Rightarrow 11-12+5 = (A+0)(-1) + 5(2)$

$4 = -A+10 \Rightarrow -6 = -A \Rightarrow \boxed{A=6}$

$$\int \frac{11x^2-12x+5}{(x^2+1)(x-2)}\,dx = \int \frac{6x}{x^2+1} + \frac{5}{x-2}\,dx$$

$u = x^2+1$
$du = 2x\,dx$

$$= \int \frac{3\,du}{u} + 5\ln|x-2| = 3\ln|x^2+1| + 5\ln|x-2| + k$$

Answer (15 points)

$\boxed{3\ln|x^2+1| + 5\ln|x-2| + k.}$

Solns

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13) $\int_3^{\infty} \frac{xdx}{\left(x^2+1\right)^2}$

$u = x^2+1$
$du = 2xdx$

$\cancel{x} = 3 \Rightarrow u = 10$
$x = \infty \Rightarrow u = \infty$

$$= \frac{1}{2}\int_{10}^{\infty} \frac{du}{u^2} = \frac{1}{2}\lim_{m\to\infty} \int_{10}^{m} \frac{du}{u^2} =$$

$$= \frac{1}{2}\lim_{m\to\infty}\left(-\frac{1}{u}\Big|_{10}^{m}\right)$$

$$= \frac{1}{2}\lim_{m\to\infty}\left(-\frac{1}{m}+\frac{1}{10}\right)$$

$$= \frac{1}{20}.$$

Answer (10 points)

$\frac{1}{20}$

SOL'NS

14) Find the area of the region in the first quadrant formed by the curve $r = \sin\theta + \cos\theta$ and the coordinate axes on the interval $\left[0, \frac{\pi}{2}\right]$.

ASSUMING THIS IS A POLAR CURVE.

FOR $0 < \theta < \pi/2$, $r > 1$.

THE AREA IS GIVEN BY $\int_0^{\pi/2} \frac{r^2}{2}\, d\theta$

$$= \int_0^{\pi/2} \frac{1}{2}(\sin\theta + \cos\theta)^2\, d\theta = \int_0^{\pi/2} \frac{1}{2}\left(\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta\right) d\theta$$

$$= \int_0^{\pi/2} \frac{1}{2} + \sin\theta\cos\theta\, d\theta \qquad \left[\int \sin\theta\cos\theta\, d\theta = \int u\, du = \frac{u^2}{2} = \frac{\sin^2\theta}{2}\right]$$

$u = \sin\theta$
$du = \cos\theta\, d\theta$

$$= \frac{1}{2}\theta + \frac{1}{2}\sin^2\theta \Big|_0^{\pi/2} = \frac{\pi}{4} + \frac{1}{2} - 0$$

Answer (15 points)

$$\frac{\pi}{4} + \frac{1}{2} = \frac{2+\pi}{4}$$