MAT126 Fall 2009

Practice Final

The actual Final exam will consist of twelve problems

Problem 1 1. Evaluate $\int_{\pi/3}^{\pi/2} \sin^3(x)\cos^2(x)dx$

2. Evaluate $\int \sin^2(x)dx$

Problem 2 1. Estimate the integral

$$\int_7^8 \frac{dx}{\ln(x)}$$

using three rectangles and

(a) right endpoints

(b) left endpoints

(c) Are your answers in 1a and 1b over- or under-estimates of the actual integral?

2. Do the same for the under-integral function $f(t) = e^{t^3}$

Problem 3 Integrate

1. $$\int \cos(\ln(x))dx$$

2. $$\int e^{\sqrt{x}}dx$$

Problem 4 Find the following indefinite integrals

1. $$\int \frac{dx}{(x+1)\sqrt{x}}$$

2. $$\int \frac{(e^x+1)dx}{e^x(e^x+2)}$$

Problem 5 Find the following indefinite integrals

1. $$\int \frac{x^2+1}{x^2-3x+2}$$

2. $$\int \frac{x^3+1}{x^2-4x+3}$$

Problem 6 1. Use trapezoidal approximation with $n = 4$ intervals of subdivision (each of equal length) to estimate the following integral

$$\int_0^{\pi} \sin^2(x)dx.$$

Estimate the error of approximation.

2. Use Simpson's rule with $n = 4$ to estimate $\int_0^1 x^5 dx$. Find the precise error of approximation.

Problem 7 1. Does the following integral converge? If yes, evaluate it:

$$\int_0^{+\infty} \exp(-t)\cos^2(t)dt$$

2. Does the following integral converge? If yes, evaluate it:

$$\int_1^{+\infty} \frac{dt}{t(t+1)}$$

3. Does the following integral converge? If yes, evaluate it:

$$\int_1^{\infty} \frac{dx}{x[\ln x]^2}$$

Problem 8 1. Find the volumes of the bodies obtained from the region enclosed by

$$y = \sin(x), 0 \leq x \leq \pi$$

and $y = 0$ by revolving about a) the x-axis, b) the line $x = -2$

2. Find the volumes of the bodies obtained from the region enclosed by

$$y = \frac{1}{x^5}, 1 \leq x < \infty,$$

$y = 0$, $x = 1$ by revolving about a) the line $y = -2$, b) the y-axis.

Problem 9 Find the length of the curve given by the graph of the function

$$y = \ln(\cos(x))$$

between points $(0, 0)$ and $(a, \ln(\cos(a))), 0 < a < \frac{\pi}{2}$.

This problem covers material (work) that we did not cover in Fall 2018, and so won't be on our exam.

Problem 10 1. A cable that weights 2lb/ft is used to lift 800lb of coal from a mineshaft 500 ft deep. Find the work done.

2. A 10 ft chain weights 25 lb and hangs from the ceiling. Find the work done in lifting the middle of the chain to the ceiling so that it is level with the upper end.**Final**

This problem covers material we did not cover in Fall 2016.

Problem 11 A granary has the shape of a half cylinder lying on its rectangular side (the cut). The cylinder's height is 10m, and the radius of the base is 2m. If the granary is full of barley, with density $600 kg/m^3$, how much work is done in removing all the grain via an opening at the top of the granary?

Problem 12 If $f(x)$ is an increasing function on $[0, 1]$, rank the following in order from least to greatest:

- f(0)
- f(1)
- The left endpoint approximation to $\int_0^1 f(x)dx$ with $n = 5$ rectangles.
- The right endpoint approximation to $\int_0^1 f(x)dx$ with $n = 5$ rectangles.
- The average value of f on $[0, 1]$.

Problem 13 For each of the following, determine if the improper integral converges or diverges. If it converges, evaluate the integral.

1. $\int_{-\infty}^{\infty} 2xe^{-x^2}dx$
2. $\int_{-\infty}^{\infty} \frac{1}{2x}e^{-x^2}dx$

This problem covers material we did not do in Fall 2018 (parametric curves).

Problem 14 A particle starts at the origin at time $t = 0$, and traces out a path given by

$$\begin{aligned} x(t) &= t \\ y(t) &= 2t^2 \end{aligned}$$

for each $t \geq 0$.

1. Express the length $l(T)$ of the path traced out by the particle from $t = 0$ to a time $t = T$ as an integral, but do not evaluate it.
2. Find $l'(T)$, the speed of the particle at time T.
3. Evaluate $l'(2)$.