① $\int (3x-1)^5\,dx$ let $u = 3x-1$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$= \frac{1}{3}\int u^5\,du$

$= \frac{1}{3}\left(\frac{u^6}{6}\right) + C = \frac{u^6}{18} + C = \frac{(3x-1)^6}{18} + C$

---

② $\int x(2-x^2)^3\,dx$ let $u = 2-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$= -\frac{1}{2}\int u^3\,du$

$= -\frac{1}{2}\left(\frac{u^4}{4}\right) + C = \frac{-u^4}{8} + C = -\frac{(2-x^2)^4}{8} + C$

---

③ $\int \sin 3x\,dx$ let $u = 3x$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$= \frac{1}{3}\int \sin u\,du$

$= \frac{-1}{3}\cos u + C = \frac{-1}{3}\cos(3x) + C$

---

④ $\int \sqrt{3x-1}\,dx$ let $u = 3x-1$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$= \frac{1}{3}\int u^{\frac{1}{2}}\,du$

$= \frac{1}{3}\,\frac{u^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{9}u^{\frac{3}{2}} + C = \frac{2}{9}(3x-1)^{\frac{3}{2}} + C$

(5) $\int x\sqrt{7x^2+12}\,dx$ let $u=7x^2+12$

$du = 14x\,dx$

$\frac{1}{14}du = x\,dx$

$= \frac{1}{14}\int u^{\frac{1}{2}}du$

$= \frac{1}{14}\ \frac{u^{\frac{3}{2}}}{\frac{3}{2}} = \frac{1}{21}u^{\frac{3}{2}} + C = \frac{1}{21}(7x^2+12)^{\frac{3}{2}} + C$

---

(6) $\int \frac{x}{(4x^2+1)^3}dx$ Let $u = 4x^2+1$

$du = 8x\,dx$

$\frac{1}{8}du = x\,dx$

$= \frac{1}{8}\int \frac{du}{u^3} = \frac{1}{8}\int u^{-3}du$

$= \frac{1}{8}\ \frac{u^{-2}}{-2} = -\frac{1}{16}\cdot\frac{1}{u^2} = -\frac{1}{16}\cdot\frac{1}{(4x^2+1)^2}\,dx$

---

(7) $\int \frac{\sin(\frac{5}{x})}{x^2}dx$ let $u = \frac{5}{x}$

$du = \frac{-5}{x^2}dx$

$-\frac{1}{5}du = \frac{dx}{x^2}$

$= -\frac{1}{5}\int \sin u\,du$

$= \frac{1}{5}\cos u + C = \frac{1}{5}\cos\left(\frac{5}{x}\right) + C$

(8) $\displaystyle\int \frac{\sec^2(\sqrt{x})}{\sqrt{x}}\,dx$ let $u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2du = \frac{dx}{\sqrt{x}}$

$= 2\displaystyle\int \sec^2 u\,du$

$= 2\tan u + C = 2\tan\sqrt{x} + C$

---

(9) $\displaystyle\int \sin(\sin\theta)\cos\theta\,d\theta$ let $u = \sin\theta$

$du = \cos\theta\,d\theta$

$= \displaystyle\int \sin u\,du = -\cos u + C = -\cos(\sin\theta) + C$

---

(10) $\displaystyle\int x\sqrt{x-3}\,dx$ let $u = x-3$

$u+3 = x$

$du = dx$

$= \displaystyle\int (u+3)\sqrt{u}\,du$

$= \displaystyle\int u^{\frac{3}{2}} + 3u^{\frac{1}{2}}\,du = \frac{u^{\frac{5}{2}}}{\frac{5}{2}} + \frac{3u^{\frac{3}{2}}}{\frac{3}{2}} + C$

$= \frac{2}{5}u^{\frac{5}{2}} + 2u^{\frac{3}{2}} + C$

$= \frac{2}{5}(x-3)^{\frac{5}{2}} + 2(x-3)^{\frac{3}{2}} + C$

(11) $\int xe^{-x}\,dx$ $\quad u = x \quad dv = e^{-x}dx$

$du = dx \quad v = -e^{-x}$

$$\int xe^{-x}dx = -xe^{-x} + \int e^{-x}dx$$

$$= -xe^{-x} - e^{-x} + C$$

---

(12) $\int \ln(2x+3)\,dx$ $\quad u = \ln(2x+3) \quad dv = dx$

$du = \frac{2}{2x+3}dx \quad v = x$

$$\int \ln(2x+3)dx = x\ln(2x+3) - \int \frac{2x}{2x+3}dx$$

$$= x\ln(2x+3) - \int\left(1 - \frac{3}{2x+3}\right)dx$$

Polynomial division

$$= x\ln(2x+3) - \left(x - \frac{3\ln(2x+3)}{2}\right) + C$$

---

(13) $\int x\ln x\,dx$ $\quad u = \ln x \quad dv = x\,dx$

$du = \frac{1}{x}dx \quad v = \frac{x^2}{2}$

$$\int x\ln x\,dx = \frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx$$

$$= \frac{x^2}{2}\ln x - \frac{1}{2}\left(\frac{x^2}{2}\right) + C$$

(14) $\int \sin^{-1}x\,dx$ $\quad u = \sin^{-1}x \quad dv = dx$

$du = \frac{1}{\sqrt{1-x^2}}\,dx \quad v = x$

$$\int \sin^{-1}x\,dx = x\sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}}\,dx$$

Let $u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$$= -\frac{1}{2}\int u^{-\frac{1}{2}}du = \frac{-u^{\frac{1}{2}}}{2\cdot\frac{1}{2}} = -u^{\frac{1}{2}}$$

$$= -\sqrt{1-x^2}$$

$$\int \sin^{-1}x\,dx = x\sin^{-1}x + \sqrt{1-x^2} + C$$

---

(15) $\int x^2e^{-x}dx$ $\quad u = x^2 \quad dv = e^{-x}dx$

$du = 2x\,dx \quad v = -e^{-x}$

$$\int x^2e^{-x}dx = -x^2e^{-x} + 2\int xe^{-x}dx$$

$u = x \quad dv = e^{-x}dx$

$du = dx \quad v = -e^{-x}$

$$= -x^2e^{-x} + 2\left(-xe^{-x} + \int e^{-x}dx\right)$$

$$= -x^2e^{-x} - 2xe^{-x} + 2\int e^{-x}dx$$

$$= -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C$$

(16) $\int e^x \sin x\,dx$ $\quad u = e^x \quad dv = \sin x\,dx$
$du = e^x dx \quad v = -\cos x$

$$\int e^x \sin x\,dx = -e^x \cos x + \int e^x \cos x\,dx$$

$u = e^x \quad dv = \cos x\,dx$
$du = e^x dx \quad v = \sin x$

$$\int e^x \sin x\,dx = -e^x \cos x + e^x \sin x - \int e^x \sin x\,dx$$

add this integral to both sides

$$2\int e^x \sin x\,dx = -e^x \cos x + e^x \sin x + C$$

$$\int e^x \sin x\,dx = \frac{-e^x \cos x + e^x \sin x}{2} + C$$

---

(17) $\int x \tan^{-1} x\,dx$ $\quad u = \tan^{-1} x \quad dv = x\,dx$
$du = \frac{1}{1+x^2}dx \quad v = \frac{x^2}{2}$

$$\int x \tan^{-1} x\,dx = \frac{x^2}{2}\tan^{-1} x + \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx$$

polynomial division

$$= \frac{x^2}{2}\tan^{-1} x + \frac{1}{2}\int 1 - \frac{1}{1+x^2}\,dx$$

$$= \frac{x^2}{2}\tan^{-1} x + \frac{1}{2}\left(x - \tan^{-1} x\right) + C$$

(19) $\int x^2 \ln x \, dx$ $\quad u = \ln x \quad dv = x^2 dx$

$du = \frac{1}{x} dx \quad v = \frac{x^3}{3}$

$$\int x^2 \ln x \, dx = \frac{x^3}{3} \ln x - \frac{1}{3}\int x^2 \, dx$$

$$= \frac{x^3}{3} \ln x - \frac{1}{3} \cdot \frac{x^3}{3} + C$$

---

(20) $\int x\cos(3x)\,dx$ $\quad u = x \quad dv = \cos(3x)dx$

$du = dx \quad v = \frac{\sin(3x)}{3}$

$$\int x\cos(3x)\,dx = \frac{x\sin(3x)}{3} - \frac{1}{3}\int \sin(3x)\,dx$$

$$= \frac{x\sin(3x)}{3} - \frac{1}{3}\left(\frac{-\cos(3x)}{3}\right) + C$$

$$= \frac{x\sin(3x)}{3} + \frac{1}{9}\cos(3x) + C$$

---

(21) $\int \cos^5 x \sin x \, dx$ $\quad$ Let $u = \cos x$

$du = -\sin x \, dx$

$$= -\int u^5 \, du = -\frac{u^6}{6} + C = -\frac{\cos^6 x}{6} + C$$

(22) $\int \sin^4 3x \cos 3x \, dx$

$u = \sin 3x$

$du = \frac{\cos 3x}{3} dx$

$3du = \cos 3x \, dx$

$= 3\int u^4 du = \frac{3u^5}{5} = \frac{3\sin^5(3x)}{5} + C$

---

(18) (oops!) $\int x\sec^2 x \, dx$

$u = x$ $\quad dv = \sec^2 x \, dx$

$du = dx$ $\quad v = \tan x$

$\int x\sec^2 x \, dx = x\tan x - \int \tan x \, dx$

$= x\tan x + \ln(\cos x) + C$

---

(23) $\int \cos^2 4x \, dx = \frac{1}{2}\int (1+\cos 8x) dx = \frac{1}{2}\left(x + \frac{\sin 8x}{8}\right) + C$

---

(24) $\int \cos^3 x \, dx = \int \cos x \cos^2 x \, dx = \int \cos x (1-\sin^2 x) dx$

$u = \sin x$

$du = \cos x \, dx$

$\int 1-u^2 du = u - \frac{u^3}{3} + C = \sin x - \frac{\sin^3 x}{3} + C$

(25) $$\int \sin^3 x \cos^3 x\,dx = \int \sin^3 x \cos^2 x \cos x\,dx$$

$$= \int \sin^3 x (1-\sin^2 x) \cos x\,dx$$

$u = \sin x$

$du = \cos x\,dx$

$$= \int u^3(1-u^2)\,du = \int u^3 - u^5\,du = \frac{u^4}{4} - \frac{u^6}{6} + C$$

$$= \frac{\sin^4 x}{4} - \frac{\sin^6 x}{6} + C$$

---

(26) $$\int \cos^2 x \sin^2 x\,dx = \int \tfrac{1}{2}(1+\cos 2x)\cdot\tfrac{1}{2}(1-\cos 2x)\,dx$$

$$= \frac{1}{4}\int (1-\cos^2 2x)\,dx = \frac{1}{4}\int \sin^2 2x\,dx = \frac{1}{4}\int \frac{1}{2}(1-\cos 4x)\,dx$$

$$= \frac{1}{8}\left(x - \frac{\sin 4x}{4}\right) + C$$

---

(27) $$\int \tan^2 x \sec^2 x\,dx$$ $u = \tan x$

$du = \sec^2 x\,dx$

$$\int u^2\,du = \frac{u^3}{3} + C$$

$$= \frac{\tan^3 x}{3} + C$$

(28) $\int \tan^3 x \sec^4 x \, dx = \int \tan^3 x \sec^2 x \sec^2 x \, dx$

$= \int \tan^3 x (1+\tan^2 x) \sec^2 x \, dx$

let $u = \tan x$

$du = \sec^2 x \, dx$

$= \int u^3 (1+u^2) du = \int u^3 + u^5 du = \frac{u^4}{4} + \frac{u^6}{6} + C$

$= \frac{\tan^4 x}{4} + \frac{\tan^6 x}{6} + C$

(29) $\int \tan^5 x \sec x \, dx = \int (\tan^2 x)^2 (\sec x \tan x) \, dx$

$= \int (\sec^2 x - 1)^2 (\sec x \tan x) dx$

$u = \sec x$

$du = \sec x \tan x \, dx$

$= \int (u^2-1)^2 du = \int u^4 - 2u^2 + 1 \; du$

$= \frac{u^5}{5} - \frac{2u^3}{3} + u + C$

$= \frac{\sec^5 x}{5} - \frac{2\sec^3 x}{3} + \sec x + C$

(30) $\int \sec^3 x\,dx$ $\quad u = \sec x \quad dv = \sec^2 x\,dx$

$du = \sec x \tan x\,dx \quad v = \tan x$

$$\int \sec^3 x\,dx = \sec x \tan x - \int \sec x \tan^2 x\,dx$$

$$\int \sec^3 x\,dx = \sec x \tan x - \int \sec x\,(\sec^2 x - 1)\,dx$$

$$\int \sec^3 x\,dx = \sec x \tan x - \int \sec^3 x\,dx + \int \sec x\,dx$$

$$2\int \sec^3 x\,dx = \sec x \tan x + \int \sec x\,dx$$

$$2\int \sec^3 x\,dx = \sec x \tan x + \ln|\sec x + \tan x| + C$$

$$\int \sec^3 x\,dx = \tfrac{1}{2}\sec x \tan x + \tfrac{1}{2}\ln|\sec x + \tan x| + C$$

(31) $$\int \frac{dx}{x^2+3x-4} = \int \frac{dx}{(x+4)(x-1)}$$

$$\frac{1}{(x+4)(x-1)} = \frac{A}{x+4} + \frac{B}{x-1}$$

$$1 = A(x-1) + B(x+4)$$

$$1 = Ax - A + Bx + 4B$$

$$1 = Ax + Bx + (-A+4B)$$

$$\cancel{A} + B = 0$$

$$-\cancel{A} + 4B = 1$$

$$5B = 1$$

$$B = \frac{1}{5} \to A = -\frac{1}{5}$$

$$\int \frac{dx}{(x+4)(x-1)} = -\frac{1}{5}\int \frac{dx}{x+4} + \frac{1}{5}\int \frac{dx}{x-1}$$

$$= -\frac{1}{5}\ln|x+4| + \frac{1}{5}\ln|x-1| + C$$

---

(32) $$\int \frac{11x+17}{2x^2+7x-4}\,dx = \int \frac{11x+17}{(2x-1)(x+4)}\,dx$$

$$\frac{11x-17}{(2x-1)(x+4)} = \frac{A}{2x-1} + \frac{B}{x+4} \to 11x-17 = A(x+4)+B(2x-1)$$

$$11x-17 = Ax+4A+2Bx-B$$

$A+2B=11 \to A+2B=11$

$4A-B=-17 \quad 8A-2B=-34$

$$9A = -23$$

$$A = \frac{-23}{9}$$

$$B = \frac{61}{9}$$

$$\int \frac{11x+17}{2x^2+7x-4}\,dx = \frac{-23}{9}\int \frac{dx}{2x-1} + \frac{61}{9}\int \frac{dx}{x+4}$$

$$= \frac{-23}{9}\,\frac{\ln|2x-1|}{2} + \frac{61}{9}\ln|x+4| + C$$

(33) $\displaystyle\int \frac{5x-5}{3x^2-8x-3}\,dx$ $\qquad \dfrac{5x-5}{3x^2-8x-3} = \dfrac{A}{3x+1} + \dfrac{B}{x-3}$

$(3x+1)(x-3)$

$5x-5 = A(x-3)+B(3x+1)$

$5x-5 = Ax-3A+3Bx+B$

$A+3B=5 \rightarrow 3A+9B=15$

$-3A+B=-5 \qquad -3A+3B=-5$

$12B=10$

$B=\frac{5}{6}$

$A=\frac{25}{18}$

$$\int \frac{5x-5}{3x^2-8x-3}\,dx = \frac{25}{18}\int\frac{dx}{3x+1} + \frac{5}{6}\int\frac{dx}{x-3}$$

$$= \frac{25}{18}\,\frac{\ln|3x+1|}{3} + \frac{5}{6}\ln|x-3| + C$$

---

(34) $\displaystyle\int \frac{2x^2-1}{(4x-1)(x^2+1)}\,dx$ $\qquad \dfrac{2x^2-1}{(4x-1)(x^2+1)} = \dfrac{A}{4x-1} + \dfrac{Bx+C}{x^2+1}$

$2x^2-1 = Ax^2+A+4Bx^2+4Cx-Bx-C$

$2x^2-1 = Ax^2+4Bx^2+4Cx-Bx+A-C$

$A+4B=2 \longrightarrow A+16C=2$

$4C-B=0 \rightarrow B=4C$

$A-C=-1 \longrightarrow A-C=-1$

$17C=3$

$C=\frac{3}{17} \quad B=\frac{12}{17} \quad A=\frac{-14}{17}$

$$\int\frac{2x^2-1}{(4x-1)(x^2+1)}\,dx = \frac{-14}{17}\int\frac{dx}{4x-1} + \frac{12}{17}\int\frac{x\,dx}{x^2+1} + \frac{3}{17}\int\frac{dx}{x^2+1}$$

$u=x^2+1$

$du=2x\,dx$

$\frac{1}{2}du=x\,dx$

$=\frac{-14}{17}\,\frac{\ln|4x-1|}{4} \qquad \frac{6}{17}\int\frac{du}{u}=\frac{6}{17}\ln|u| \qquad +\frac{3}{17}\operatorname{Tan}^{-1}x + C$

$$= \frac{-14}{4}\ln|4x-1| + 6\ln|x^2+1| + \frac{3}{2}\operatorname{Tan}^{-1}x + C$$

35 $\int \frac{dx}{x(x^2+1)}$ $\quad \frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

$1 = Ax^2 + A + Bx^2 + Cx$

$A+B=0$

$C=0$

$A=1$

$B=-1$

$$\int \frac{dx}{x(x^2+1)} = \int \frac{dx}{x} - \int \frac{x}{x^2+1}\, dx$$

$= \ln|x| -$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2} du = x\,dx$

$$\frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| = \frac{1}{2}\ln|x^2+1|$$

$$= \ln|x| - \frac{1}{2}\ln|x^2+1| + C$$