1a $f(x) = \int (3x^2 - 5x + 1)\,dx = x^3 - \frac{5x^2}{2} + x + c$

$10 = 0 + c$

$c = 10$

$f(x) = x^3 - \frac{5x^2}{2} + x + 10$

---

b) $f(y) = \int 4y^3 + y^2 + 6y\,dy = y^4 + \frac{y^3}{3} + 3y^2 + c$

$0 = 1 + \frac{1}{3} + 3 + c$

$c = -\frac{13}{3}$

$f(y) = y^4 + \frac{y^3}{3} + 3y^2 - \frac{13}{3}$

---

c) $f(\theta) = \int \sin(\frac{\pi}{4}\theta) + \frac{2}{\theta}\,d\theta = \frac{-\cos(\frac{\pi}{4}\theta)}{\frac{\pi}{4}} + 2\ln\theta + c$

$= \frac{-4\cos(\frac{\pi}{4}\theta)}{\pi} + 2\ln\theta + C$

$5 = \frac{-4\cos\frac{\pi}{4}}{\pi} + 2\ln 1 + c$

$5 = \frac{-4(\frac{\sqrt{2}}{2})}{\pi} + c \Rightarrow c = 5 + \frac{2\sqrt{2}}{\pi}$

$f(\theta) = \frac{-4\cos(\frac{\pi}{4}\theta)}{\pi} + 2\ln\theta + 5 + \frac{2\sqrt{2}}{\pi}$

---

d) $f(x) = \int \frac{e^x - e^{-x}}{2}\,dx = \frac{e^x + e^{-x}}{2} + C$

$1 = \frac{e^0 + e^0}{2} + C$

$C = 0$

$f(x) = \frac{e^x + e^{-x}}{2}$

e) $f(z) = \int 3z^2 - \frac{1}{\sqrt{z}}\, dz = \int 3z^2 - z^{-\frac{1}{2}}\, dz = z^3 - 2\sqrt{z} + C$ ②

$4 = 1 - 2 + C$

$C = 5$

$f(z) = z^3 - 2\sqrt{z} + 5$

2a) $f'(x) = 5 + x^2$

b) $f'(3) = 5 + 3^2 = 14$

c) $f(3) = \int_2^3 (5 + t^2)\, dt = 5t + \frac{t^3}{3} \Big|_2^3 = (15 + 9) - (10 + \frac{8}{3}) = \frac{34}{3}$

---

b) $g'(x) = \sin x$

$g''(x) = \cos x$

---

c) $f'(x) = \sqrt{1 - \sin^2 x}\,(\cos x) - \sqrt{1 - \cos^2 x}\,(-\sin x)$

$= \sqrt{\cos^2 x}\, \cos x + \sqrt{\sin^2 x}\, \sin x$

$= \cos^2 x + \sin^2 x = 1$

$f'(3) = 1$, $f'(\frac{\pi}{4}) = 1$

---

d) $h(x) = \int_{x^2}^{0} (t^3 - 1)\, dt = -\int_0^{x^2} (t^3 - 1)\, dt \quad h'(x) = (x^6 - 1)\, 2x$

$h(0) = \int_0^0 (t^3 - 1)\, dt = 0$

$h(1) = -\int_0^1 (t^3 - 1)\, dt = -\left(\frac{t^4}{4} - t\right) \Big|_0^1 = -\left(-\frac{3}{4} - 0\right) = \frac{3}{4}$

e) $f'(x) = \tan(x^2)(2x)$

$f'(\sqrt{\pi}) = \tan\pi\,(2\sqrt{\pi}) = 0$

3a) $\displaystyle\int_1^2 x^2+4x+\sqrt{x}\,dx = \frac{x^3}{3}+2x^2+\frac{2}{3}x^{\frac{3}{2}}\Big|_1^2$

$= \left[\frac{8}{3}+8+\frac{2}{3}(2^{3/2})\right]-\left[\frac{1}{3}+2+\frac{2}{3}\right]$

b) $\displaystyle\int e^x \sec^2 x\,dx = e^x - \tan x + C$

c) $\displaystyle\int_1^{e^3} \frac{4}{x}\,dx = 4\ln|x|\,\Big|_1^{e^3} = 4(3)-0 = 12$

d) $\displaystyle\int \sec x\sqrt{1+\tan^2 x}\,dx = \int \sec x\sqrt{\sec^2 x}\,dx$

$\displaystyle = \int \sec x \cdot \sec x\,dx = \int \sec^2 x\,dx = \tan x + C$

e) $\displaystyle\int_1^4 \pi^2\,dx = \pi^2 x\Big|_1^4 = 4\pi^2 - \pi^2 = 3\pi^2$

f) $\int \cos^9 x \sin x \, dx$ let $u = \cos x$

$du = -\sin x \, dx$

$= -\int u^9 du = -\frac{u^{10}}{10} + C = -\frac{\cos^{10} x}{10} + C$

---

g) $\int x e^{x^2} dx$ let $u = x^2$

$du = 2x \, dx$

$\frac{1}{2} du = x \, dx$

$= \frac{1}{2}\int e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C$

---

h) $\int \frac{2x \, dx}{x^2+1}$ Let $u = x^2+1$

$du = 2x \, dx$

$= \int \frac{du}{u} = \ln|u| + C = \ln(x^2+1) + C$

---

i) $\int \frac{\cos\sqrt{x}}{\sqrt{x}} dx$ let $u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}} dx$

$2du = \frac{dx}{\sqrt{x}}$

$= 2\int \cos u \, du = 2\sin u + C = 2\sin\sqrt{x} + C$

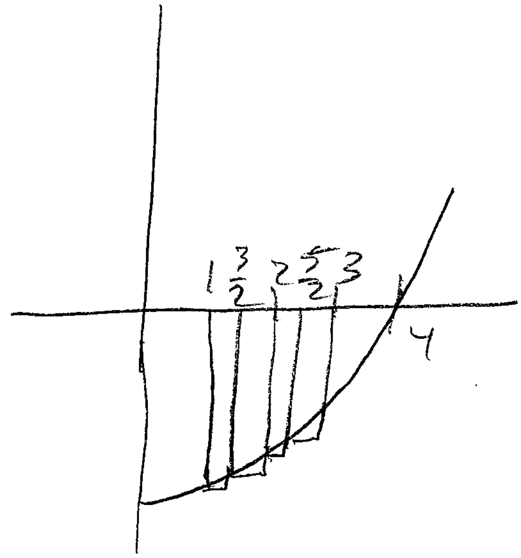
---

j) $\int \tan^3 x \sec^2 x \, dx$ let $u = \tan x$

$du = \sec^2 x \, dx$

$= \int u^3 du = \frac{u^4}{4} + C = \frac{\tan^4 x}{4} + C$

4a



i) $A = \frac{1}{2}\left[f(1) + f(\tfrac{3}{2}) + f(2) + f(\tfrac{5}{2})\right]$

$= \frac{1}{2}\left[-15 - \frac{55}{4} - 12 - \frac{39}{4}\right] = -\frac{101}{4}$

ii) $A = \frac{2}{n}\sum_{i=1}^{n}\left(1+\frac{2i}{n}\right)^2 - 16$

iii) $\frac{2}{n}\sum\left(1+\frac{4i}{n}+\frac{4i^2}{n^2}\right) - 16 = \frac{2}{n}\sum -15 + \frac{4i}{n} + \frac{4i^2}{n^2}$

$= \frac{2}{n}\sum_{i=1}^{n} -15 + \frac{2}{n}\sum_{i=1}^{n}\frac{4i}{n} + \frac{2}{n}\sum_{i=1}^{n}\frac{4i^2}{n^2}$

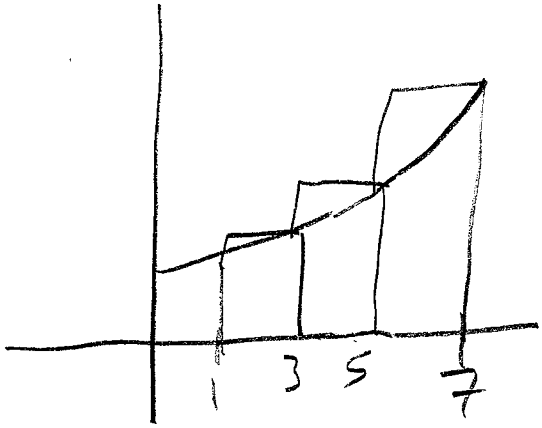
$= \frac{2}{n}\sum_{i=1}^{n} -15 + \frac{8}{n^2}\sum_{i=1}^{n} i + \frac{8}{n^3}\sum_{i=1}^{n} i^2$

$= \frac{2}{n}(-15n) + \frac{8}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{8}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$

$\lim_{n\to\infty} = -30 + 4 + \frac{16}{6} = -\frac{70}{3}$

OR $\int_1^3 x^2 - 16\, dx = \frac{x^3}{3} - 16x \Big|_1^3 = -\frac{70}{3}$

b)



i) $A = 2\left[f(3) + f(5) + f(7)\right]$

$= 2\left[31 + 129 + 347\right] = 1014$

ii) $A = \frac{6}{n}\sum_{i=1}^{n}\left(1 + \frac{6i}{n}\right)^3 + 4$

iii) $A = \frac{6}{n}\sum_{i=1}^{n} 1 + 3\left(\frac{6i}{n}\right) + 3\left(\frac{36i^2}{n^2}\right) + \frac{216i^3}{n^3} + 4$

$= \frac{6}{n}\sum_{i=1}^{n} 5 + \frac{18i}{n} + \frac{108i^2}{n^2} + \frac{216}{n^3}i^3$

$= \frac{6}{n}\sum_{i=1}^{n} 5 + \frac{108}{n^2}\sum_{i=1}^{n} i + \frac{648}{n^3}\sum_{i=1}^{n} i^2 + \frac{1296}{n^4}\sum_{i=1}^{n} i^3$

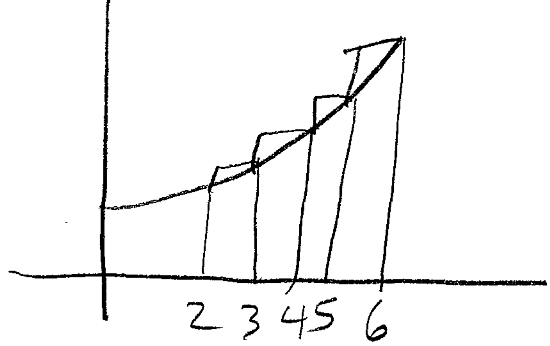
$= \frac{6}{n}(5n) + \frac{108}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{648}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) + \frac{1296}{n^4}\left(\frac{n(n+1)}{2}\right)^2$

$= 30 + 54\left(\frac{n+1}{n}\right) + 108\frac{(n+1)(2n+1)}{n^2} + \frac{324}{n^2}(n+1)^2$

$\lim_{n \to \infty} = 30 + 54 + 216 + 324 = 624$

OR $\int_1^7 x^3 + 4\,dx = \frac{x^4}{4} + 4x \Big|_1^7 = 624$

(c)



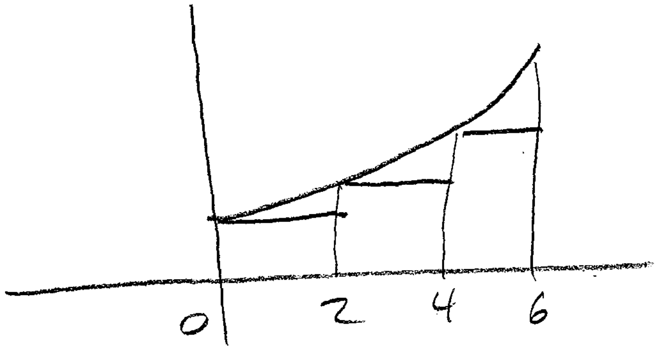
i) $A = 1\left[f(3)+f(4)+f(5)+f(6)\right]$

$= 1+e^3+1+e^4\ 1+e^5+1+e^6 = 4+e^3+e^4+e^5+e^6$

ii) $A = \frac{4}{n}\sum_{i=1}^{n} 1+e^{(2+\frac{4}{n}i)}$

iii) $\int_2^6 1+e^x\,dx = x+e^x\Big|_2^6 = (6+e^6)-(2+e^2) = 4+e^2+e^6$

(d)



i) $A = 2[f(0)+f(2)+f(4)]$

$= 2[1+7+21] = 58$

ii) $A = \frac{6}{n}\sum_{i=1}^{n}\left(\frac{6i}{n}\right)^2 + \frac{6i}{n} + 1 = \frac{6}{n}\sum_{i=1}^{n}\frac{36i^2}{n^2} + \frac{6i}{n} + 1$

iii) $A = \frac{216}{n^3}\sum_{i=1}^{n} i^2 + \frac{36}{n^2}\sum_{i=1}^{n} i + \frac{6}{n}\sum_{i=1}^{n} 1$

$= \frac{216}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) + \frac{36}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{6}{n}(n)$

$\frac{36}{n^2}(n+1)(2n+1) + 18\left(\frac{n+1}{n}\right) + 6$

$\lim_{n\to\infty} = 72+18+6 = 96$

$\int_0^6 (x^2+x+1)\,dx = \frac{x^3}{3}+\frac{x^2}{2}+x\Big|_0^6 = (72+18+6)-0) = 96$