MAT126 Final Exam

150 minutes; 9:30am-12:00pm

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Summer II 2016

Name and Solar ID: ____________________

Date: ____________________

Be sure to show all of your work for each problem. For calculations, unless otherwise stated, try and simplify as much as possible.

Grade Table (for Instructor use only)

Question	Points	Bonus Points	Score
1	20	0	
2	20	0	
3	20	0	
4	20	0	
5	20	0	
6	20	0	
7	20	0	
8	20	0	
9	30	0	
10	20	0	
11	0	20	
12	0	20	
Total:	210	40	

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1. (20 points) Write the following as a definite integral and then solve:

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{16 - (x_i^*)^2}\, \Delta x$$

where $x_i^* = 0 + \frac{4i}{n}$ and $\Delta x = \frac{4}{n}$.

2. (20 points) Compute the following:

$$\frac{d}{dx}(\int_{x^2}^{\frac{1}{x}} e^{(v^3+7)^4}\ dv)$$

3. (20 points) Solve the following integral:

$$\int w^4 e^{-w^5} (e^{-w^5} + 7)^3\ dw$$

4. (20 points) Compute the following:

$$\int \frac{\cos\theta}{(3+\sin\theta)^2(4+\sin\theta)}\,d\theta$$

(HINT: Let $u = \sin(\theta)$ and proceed from there)

5. (20 points) Please state whether the following integrals are convergent or divergent. If convergent, state the value for which the integral ultimately converges to. You must justify your answers!

   (a) $\int_1^\infty z^7 \ln(z) dz$

   (b) $\int_0^{\frac{\pi}{4}} \sec^2(2u) \tan^2(2u)\, du$

6. (20 points) Find the following information for the region bounded by $y = 3 + x^2$, $y = 2 - x^2$, $x = -1$ and $x = 1$:

   (a) Sketch the region.

   (b) Find the area of the region.

   (c) Sketch the disk, washer, or shell that we'd obtain if we rotated our region about the line $x = 2$.

   (d) Find the volume of the solid from part (c).

7. (20 points) Set-up, but DO NOT EVALUATE, the integral that would give us the arc length of

$$x = \ln(\tan(e^{-y}))$$

on the interval $2 \leq y \leq 4$.

8. (20 points) Find the average of

$$f(x) = \frac{x^2}{\sqrt{x^3+1}}$$

on the interval $[1, 3]$. You may leave your final answer in radical form.

9. (30 points) Please complete the following:

(a) (10 points) A force of $300N$ stretches a spring $30cm$ from its natural length. How much work is done in stretching the spring from $50cm$ from its natural length to $60cm$ from its natural length?

(b) (20 points) Find the centroid of the region bounded by $y = 2x - x^2$ and $y = 0$.

10. (20 points) We are given the probability density function for a random variable X as such:

$$f(x) = cx^2 e^{-\frac{1}{2}x}, \quad x \in [0, \infty)$$

and 0 otherwise.

(a) Find the value of c that makes $f(x)$ a probability density function.

(b) What is the $P(0 \leq X \leq 1)$? (You may leave your answer in terms of e)

11. (20 points (bonus)) Prove that, if $n \geq 1$,

$$\int \sin^n(x)dx = -\frac{1}{n}\cos(x)\sin^{n-1}(x) + \frac{n-1}{n}\int \sin^{n-2}(x)dx$$

12. (20 points (bonus)) Prove that, if $n \geq 1$ is an integer,

$$\int_0^\infty x^n e^{-x} dx = n!$$

(saying "by definition of the gamma function" does not count as proof)