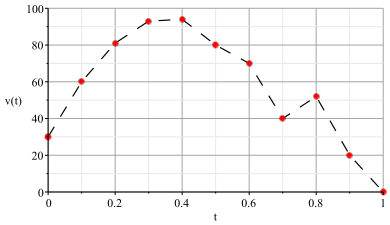
MAT126, Paper Homework "Nab"

1. After being kidnapped on a trip to Canada, little Jimmy was able to see the speedometer of the kidnapper's car, and carefully noted down the speeds (in km/hr) every tenth of an hour (six minutes). Jimmy was able to send these speeds to his friend Juan via a message tied to the leg of a pigeon. Juan wants to figure out how far away Jimmy is from where they nabbed him.

time t	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1
speed $v(t)$	30	60	81	93	94	80	70	40	52	20	0



Using the midpoint rule, estimate the total distance (in km) that Jimmy was taken, that is, estimate the integral $\int_0^1 v(t)\,dt$. (Use only the data provided– don't average to guess at times not listed.)

Using the assumption $-10\frac{km}{hr^2} \le v''(t) \le 10\frac{km}{hr^2}$, determine the maximum error in your estimate.

2. Approximate $\displaystyle\int_0^1 \ln(x)\cos(x)\,dx$ using Simpson's rule with $n = 4$ intervals (that is, use 5 data points), and give an estimate on the error of your approximation.

   You can get around the problem of $\ln(0)\cos(0)$ not being defined as follows. First integrate by parts, and then it is reasonable to take $\sin(0)/0 = 1$ since $\lim_{x\to 0}\frac{\sin(x)}{x} = 1$.

   Similarly, you can use L'Hospital's rule to determine that $\lim_{x\to 0^+}\ln(x)\sin(x) = 0$.

   So that you don't kill yourself taking derivatives, I'll tell you that the 4th derivative of $\sin(x)/x$ is $((x^4 - 12x^2 + 24)\sin(x) + (4x^3 - 24)\cos(x))/x^5$, which is between $-1/5$ and $1/5$ for any value of x.