Stony Brook University MAT 125 – Calculus A

Dept. of Mathematics **Final Exam** December 14, 2016

Please show all of your work.

1) Evaluate the following limits:

a) $\lim_{x\to 6}\frac{x^2-8x+12}{x-6}=$ $\frac{0}{0}$ so $\Rightarrow$ $\lim_{x\to 6}\frac{(x-2)(x-6)}{(x-6)}=$

$\lim_{x\to 6} x-2 = 4$

Answer (2 points)

b) $\lim_{x\to 0}\frac{5\sin 2x}{x}=$ $5\lim_{x\to 0}\frac{\sin 2x}{x}$ set $y=2x \Rightarrow x=\frac{y}{2}$

as $x\to 0, y\to 0$ so we have $5\lim_{y\to 0}\frac{\sin y}{\frac{y}{2}} = 5\cdot 2\lim_{y\to 0}\frac{\sin y}{y}$

$= 10\cdot 1 = 10$

Answer (2 points)

Final Exam

Please show all of your work.

2) Find $f'(x)$ if $f(x) = 5\cos^2(x) + 4\sin^2(x)$

$f(x) = 5(\cos x)^2 + 4(\sin x)^2 \Rightarrow f'(x) = 5 \cdot 2(\cos x)' \cdot (-\sin x)$
$+ 4 \cdot 2 (\sin x)(\cos x)$

$= -10 \cos x \sin x + 8 \sin x \cos x$
$= -10 \cos x \sin x + 8 \cos x \sin x = -2 \cos x \sin x$

Answer (4 points)

3) Find $f'(x)$ if $f(x) = \frac{5}{x} - 2\sqrt{x} + e^2$

$f(x) = 5x^{-1} - 2x^{\frac{1}{2}} + e^2$
$f'(x) = -5x^{-2} - x^{-\frac{1}{2}} + 0 = -\frac{5}{x^2} - \frac{1}{\sqrt{x}}$

You could stop here or do ↓

$-\frac{5}{x^2} - \frac{1}{x^{\frac{1}{2}}} \cdot \frac{x^{3/2}}{x^{3/2}} = \frac{-5}{x^2} - \frac{x^{\frac{3}{2}}}{x^2} = -\frac{5 + x^{\frac{3}{2}}}{x^2}$

Answer (4 points)

Please show all of your work

4) Find $f'(x)$ if $f(x)=\dfrac{3x^2+1}{1-x^2}$ $\Rightarrow f'(x) = \dfrac{(1-x^2)(6x)-(3x^2+1)(-2x)}{(1-x^2)^2}$

$= \dfrac{6x-6x^3-[-6x^3-2x]}{(1-x^2)^2} = \dfrac{6x-6x^3+6x^3+2x}{(1-x^2)^2}$

$= \dfrac{8x}{(1-x^2)^2}$

Answer (4 points)

5) Find $f'(x)$ if $f(x)=\arctan\left(\sqrt{x}\right)$:

$f'(x) = \dfrac{1}{1+(\sqrt{x})^2} \cdot \dfrac{d}{dx}\left(x^{\frac{1}{2}}\right) = \dfrac{1}{1+x} \cdot \dfrac{1}{2}x^{-\frac{1}{2}} = \dfrac{1}{2(1+x)\sqrt{x}}$

Answer (4 points)

6) Find $\frac{dy}{dx}$ if $4x^2 + 3y^2 - 8 = x^3 + 5y^3$

Implicit disserentiation: $8x + 6y\frac{dy}{dx} + 0 = 3x^2 + 15y^2\frac{dy}{dx}$

$\Rightarrow 6y\frac{dy}{dx} - 15y^2\frac{dy}{dx} = -8x + 3x^2$

$(6y - 15y^2)\frac{dy}{dx} = 3x^2 - 8x \Rightarrow \frac{dy}{dx} = \frac{3x^2 - 8x}{6y - 15y^2}$.

or $\frac{-3x^2 + 8x}{-6y + 15y^2}$

Answer (4 points)

7) Find $\frac{dy}{dx}$ if $y = xe^{2x}$

$\frac{dy}{dx} = x\,e^{2x} \cdot 2 + e^{2x} \cdot 1$

$= 2xe^{2x} + e^{2x}$

$= (2x+1)e^{2x}$

Answer (4 points)

8) Find $\dfrac{dy}{dx}$ if $y = \ln(1 + \tan x)$

$$\frac{dy}{dx} = \frac{1}{1+\tan x} \cdot \frac{0 + \sec^2 x}{1} = \frac{\sec^2 x}{1 + \tan x}.$$

Answer (4 points)