MAT 125-Final Exam Part 1-Fall 2015-Lecture 3

NAME: SOLUTIONS

TA Name:________________

When do you have recitation? DUNNO, I NEVER GO.

*This exam is pass/fail. You must score 9 out of 12 to pass.

1. Find the absolute maximum of $f(x) = 2x^2 - 8x$ for $0 \leq x \leq 3$.

FIND CRIT PTS:

$f'(x) = 4x - 8$, so $f'(2) = 0$.

CHECK:

$f(0) = 0$

$f(2) = 8 - 16 = -8$

$f(3) = 18 - 24 = -5$.

THUS, ABS MAX IS AT $(0, 0)$.

2. Find the derivative of $\sqrt{3 + \cos x} - \ln(\ln x - 2)$ $= (3+\cos x)^{1/2} - \ln(\ln x - 2)$

USING CHAIN RULE, GET

$$\frac{1}{2}(3+\cos x)(-\sin x) - \frac{1}{\ln x - 2} \cdot \frac{1}{x}$$

$$= -\frac{3\sin x + \sin x \cos x}{2} - \frac{1}{x(\ln x - 2)}$$

EITHER IS FINE.

3. Find the x value of the inflection point for $f(x) = 7 - 3x - 5x^3$.

$f'(x) = -3 - 15x^2$

$f''(x) = -30x$

$f''(x) = 0$ IF $X = 0$.

NOTE $f''(x) > 0$ FOR $X < 0$

AND $f''(x) < 0$ FOR $X > 0$,

SO $\boxed{X=0}$ IS AN INFLECTION POINT.

4. Find the derivative of $(2x + 1)^{31} \sin x$.

USING CHAIN RULE AND PRODUCT RULE:

$$31(2x+1)^{30} \cdot 2 \cdot \sin x + (2x+1)^{31} \cos x$$

$$= (2x+1)^{30}\left(62 \sin x + (2x+1)\cos x\right)$$

5. Determine the x value of any critcical points for $f(x) = \frac{x^3}{3} - 25x$.

$f'(x) = x^2 - 25$.

$f'(x) = 0$ IF $X = \pm 5$.

CRITICAL POINTS ARE $X = 5$, $X = -5$.

6.Write the equation of the tangent line to $y = \tan x$ at $x = 0$.

$y' = \sec^2 x$

AT $X=0$, SLOPE IS $\sec^2(0) = \frac{1}{\cos^2(0)} = 1$

WHEN $X=0$, $\tan x = 0$.

SO LINE IS

$y = x$.

7.Determine the horizontal asymptote of $y = \frac{80x+e^x}{8x+e^x}$

$$\lim_{x\to+\infty} \frac{80x+e^x}{8x+e^x} = \lim_{x\to+\infty} \frac{e^x}{e^x} = 1$$

$$\lim_{x\to-\infty} \frac{80x+e^x}{8x+e^x} = \frac{80}{8} = 10$$

THERE ARE TWO HORIZONTAL ASYPTOTES:

$X=1$ (ON THE RIGHT) AND $X=10$ (ON THE LEFT)

[THERE IS ALSO A VERTICAL AYSMPT. WHEN $e^x = -8x$, AROUND $X = -0.1178\ldots$]

8.If the position of a particle is given by $e^{\sin t}$, find the acceleration.

$S(t) = e^{\sin t}$

$V(t) = S'(t) = \cos t \cdot e^{\sin t}$

$A(t) = S''(t) = -\sin t\, e^{\sin t} + \sin t \cos t\, e^{\sin t}$

9.If $a(t) = \frac{-1}{\sqrt{1-t^2}}$ is the acceleration of an object, find its velocity if $V(0) = 3$. [WONT BE ON TEST DURING APRIL CUZ HAVEN'T COVERED]

ANTI DERIVATIVE OF $\frac{-1}{\sqrt{1-t^2}}$ IS $-\arcsin t + C = V(t)$.

IF $V(0) = 3$, THEN $3 = -\arcsin(0) + C$, SO $C = 3$.

$\therefore V(t) = 3 - \arcsin t$.

10.If $f'(x) = -x^2 - 1$, on what interval(s) is f concave down?

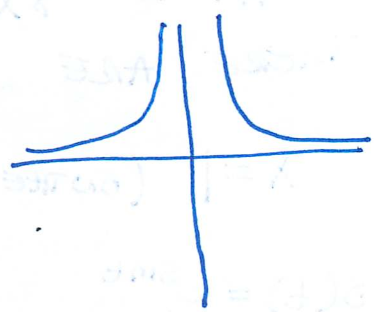
$f''(x) = -2x$, SO f WILL BE CONCAVE DOWN WHEN $-2x < 0$, IE FOR $X > 0$. OR $(0, \infty)$ IF YOU PREFER.

11.Draw a graph of $y = x^{-\frac{8}{5}}$.

$f(x) = X^{-8/5} = \frac{1}{\sqrt[5]{x^8}}$, UNDEFINED FOR $X = 0$.

$f'(x) = -\frac{8}{5} X^{-3/5}$, SO INCR FOR $X < 0$, DECR FOR $X > 0$

$f''(x) = \frac{24}{25} X^{-8/5}$, ALWAYS POSITIVE, SO ALWAYS CONC. UP.



12.Draw a graph of any antiderivative of e^{-x}.

(NOT ON APRIL EXAM).

ANTIDERIV. OF e^{-x} IS $-e^{-x} + C$.

FOR $C = 0$, LOOKS LIKE