# MAT 125       Second Midterm

**November 17, 2008**

**Name:** _____    **ID:** _____    **Rec:** _____

| Question: | 1 | 2 | 3 | 4 | 5 | 6 | Total |
|-----------|----|----|----|----|----|----|-------|
| Points:   | 12 | 12 | 10 | 10 | 12 | 10 | 66    |
| Score:    |    |    |    |    |    |    |       |

There are 6 problems in this exam, printed on 6 pages (not including this cover sheet). Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate **clearly** what is where if you expect someone to look at it. **Books, calculators, extra papers, and discussions with friends are not permitted.** You are free to use a summon the spirit of Gottfried Wilhelm Leibniz to help you, although he might speak to you in archaic German.

Leave all answers in exact form (that is, do *not* approximate $\pi$, square roots, and so on.)

You have 90 minutes to complete this exam.

1. For each of the functions $f(x)$ given below, find $f'(x)$).

   (a) $\boxed{3 \text{ points}}$ $f(x) = x^9 + 5x^4 + 3x^2 + \pi^2$

   (b) $\boxed{3 \text{ points}}$ $f(x) = \cos(x)\sin(2x)$

   (c) $\boxed{3 \text{ points}}$ $f(x) = \dfrac{\sin(x)}{\cos(x)}$

   (d) $\boxed{3 \text{ points}}$ $f(x) = \arctan\left(x^4\right)$

       This uses material that won't be on our exam, but will be on the final

2. Compute each of the following derivatives as indicated:

(a) $\boxed{3 \text{ points}}$ $\dfrac{d}{dt}\left[\dfrac{e^t - e^{-t}}{e^t + e^{-t}}\right]$

(b) $\boxed{3 \text{ points}}$ $\dfrac{d}{du}\left[\, u\ln(u)\,\right]$

(c) $\boxed{3 \text{ points}}$ $\dfrac{d}{dz}\left[\,\ln(\sec(7z))\,\right]$

(d) $\boxed{3 \text{ points}}$ $\dfrac{d}{dx}\left[\,e^x - x^e\,\right]$
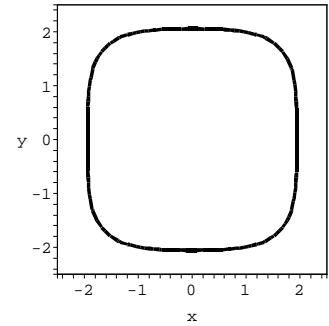
3. ☐ 10 points ☐ Let $\mathcal{C}$ be the curve which consists of the set of points for which
$$x^4 + x^2 + y^4 = 18$$
(see the graph at right).

Write the equation of the line tangent to $\mathcal{C}$ which passes through the point $(-1, 2)$.

4. 10 points  Give the $x$ and $y$ coordinates of the (absolute) maximum and minimum values of the function

$$y = x^4 - 8x^2 - 1 \qquad \text{where} \qquad -3 \le x \le 1.$$

This problem uses material we have not yet covered, and so won't be on the midterm.

5. Let $f(x) = x\, e^{-2x}$.

   (a) $\boxed{3 \text{ points}}$ Calculate $f'(x)$

   (b) $\boxed{3 \text{ points}}$ Calculate $f''(x)$

   (c) $\boxed{3 \text{ points}}$ For what values of $x$ is $f(x)$ increasing?

   (d) $\boxed{3 \text{ points}}$ For what values of $x$ is $f(x)$ concave down?

6. $\boxed{10\ \text{points}}$ A leaky oil tanker is anchored offshore. Because the water is very calm, the oil slick always stays circular as it expands, with a uniform depth of 1 meter. If the oil is leaking from the tanker at a rate of $100\ \frac{m^3}{hr}$, how fast is the radius of the slick expanding (in $\frac{m}{hr}$) when the diameter is 10 meters?

This problem uses material we have not yet covered in class, and so won't be on the midterm.