MAT125

Sample Midterm #2

Problem #1: Find the derivative of each function.

a) $f(x) = 4x^3 + x - 7$

$f'(x) = 12x^2 + 1$

b) $f(x) = \frac{x+x^2}{4x-11}$

$f'(x) = \frac{(4x-11)(1+2x) - (x+x^2)(4)}{(4x-11)^2}$

c) $f(x) = (5x^2 - x)(11x + \sqrt{x})$

$f'(x) = (5x^2-x)\left(11 + \frac{1}{2\sqrt{x}}\right) + (11x+\sqrt{x})(10x-1)$

d) $f(x) = \tan x - 3\csc x$

$f'(x) = \sec^2 x - 3(-\csc x \cot x)$

$= \sec^2 x + 3\csc x \cot x$

e) $f(x) = \ln \frac{(2x+5)^4}{(x-3)^2}$ $= \ln(2x+5)^4 - \ln(x-3)^2$

$= 4\ln(2x+5) - 2\ln(x-3)$

$f'(x) = 4\left(\frac{2}{2x+5}\right) - 2\left(\frac{1}{x-3}\right)$

Problem #2: Find the equation of the tangent line to y= $7x^2 - \frac{9}{x}$ at $x = 1$.

$$y = 7(1^2) - \frac{9}{1} = -2$$

$$y - (-2) = m(x-1)$$

$$\frac{dy}{dx} = 14x + \frac{9}{x^2}$$

at $x=1$: $14(1) + \frac{9}{1^2} = 23$

$$y + 2 = 23(x-1)$$

Problem #3. Find all values of x where $y = x^3 - 3x^2 - 24x + 2$ has an absolute maximum or minimum on the interval $[-3, 10]$.

f'(x) = $3x^{1}$ - 6x -24 = 3(x-4)(x+2)

So, there are critical points at x=-2 and x=4
(that is, when y' = 0)

These, and the two endpoints, are possible extreme values.
Let's see which is which:

f(-3) = -27 - 3*9   24*3   2 =   20
f(-2) = -8 - 12   48   2 = 30
f(4) = 64 - 48 - 24*4   2 = -78
f(10)= 1000 - 300 - 240   2 = 462

So, the absolute maximum on [-3,10] occurs at x=10 and y=462
and the absolute minimum on [-3,10] occurs at x=-2 and y=30.

Problem #4: Find $\frac{dy}{dx}$ if $3x^2 + xy - y^4 = 1$.

$$6x + \left(x\frac{dy}{dx} + y(1)\right) - 4y^3\frac{dy}{dx} = 0$$

$$6x + x\frac{dy}{dx} + y - 4y^3\frac{dy}{dx} = 0$$

$$6x + y = 4y^3\frac{dy}{dx} - x\frac{dy}{dx}$$

$$6x + y = \frac{dy}{dx}(4y^3 - x)$$

$$\frac{6x+y}{4y^3-x} = \frac{dy}{dx}$$

Problem #5: Find the equation of the tangent line to $2\sin x - \cos y = \sqrt{2}$ at $(\frac{\pi}{4}, \frac{\pi}{2})$.

$$2\cos x + \sin y \frac{dy}{dx} = 0$$

$$2\cos \frac{\pi}{4} + \sin \frac{\pi}{2} \frac{dy}{dx} = 0$$

$$2\left(\frac{\sqrt{2}}{2}\right) + (1) \frac{dy}{dx} = 0$$

$$\sqrt{2} = -1 \frac{dy}{dx}$$

$$-\sqrt{2} = \frac{dy}{dx}$$

$$y - \frac{\pi}{2} = -\sqrt{2}\left(x - \frac{\pi}{4}\right)$$

Problem #6: Find $\frac{dy}{dx}$ if $y = \tan^{-1}(x-1)$

$$\frac{dy}{dx} = \frac{1}{1+(x-1)^2}(1) = \frac{1}{1+(x-1)^2}$$

Problem #7. Find all x-values of $f(x) = x^{1/3} - \dfrac{x^{4/3}}{8}$ for which either $f'(x) = 0$ or $f'(x)$ is not defined.

$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} - \frac{1}{8} \cdot \frac{4}{3} x^{\frac{1}{3}}$$

$$= \frac{1}{3\sqrt[3]{x^2}} - \frac{\sqrt[3]{x}}{6} = 0$$

$$\frac{1}{3\sqrt[3]{x^2}} = \frac{\sqrt[3]{x}}{6}$$

cross multiply: $6 = 3x$

$x = 2$ is where $f'(x) = 0$

$f'(x)$ is undefined where $x = 0$

Problem #8: Find $\frac{dy}{dx}$:

a) $x^2y^2 - 4y^3 = 1$

$$x^2\left(2y\frac{dy}{dx}\right) + 2xy^2 - 12y^2\frac{dy}{dx} = 0$$

$$2x^2y\frac{dy}{dx} - 12y^2\frac{dy}{dx} = -2xy^2$$

$$\frac{dy}{dx}(2x^2y - 12y^2) = -2xy^2$$

$$\frac{dy}{dx} = \frac{-2xy^2}{2x^2y - 12y^2} = \frac{2xy^2}{12y^2 - 2x^2y}$$

b) $y = xe^x$

$$\frac{dy}{dx} = xe^x + (1)e^x = xe^x + e^x = e^x(x+1)$$