Limits

(1) $= 27$

(2) $= \frac{6}{28}$

(3) $= 2$

(4) $= 1$

(5) $= \frac{9}{2}$

(6) $= \infty$

(7) $= -\infty$

(8) = DNE

(9) $= \frac{4}{9}$

(10) $= \frac{4}{9}$ (same problem - sorry!)

(11) $= 0$

(12) $= 0$ (same problem - oops!)

(13) = DNE

(14) $= \frac{3}{5}$

(15) $= 0$

(16) $= \frac{3}{5}$

(17) $= \frac{2}{7}$

(18) $= \frac{2}{21}$

(19) $= 6$

(20) $= 48$

Continuity

(1) $\lim_{x \to 3^-} f(x) = -2 \quad \lim_{x \to 3^+} f(x) = 11 + 3a$

$$\text{so } 11 + 3a = -2 \rightarrow a = -\frac{13}{3}$$

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(2) $\lim_{x \to 2^-} f(x) = 4b + 8 \quad \lim_{x \to 2^+} f(x) = 13 - 2b$

$$\text{so } 4b + 8 = 13 - 2b \rightarrow b = \frac{5}{6}$$

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(3) $\lim_{x \to 1^-} f(x) = a + 2b - 4 \quad \lim_{x \to 1^+} f(x) = 1 + 4a + b$

$$\text{so } a + 2b - 4 = 1 + 4a + b$$

$\lim_{x \to 2^-} f(x) = 4 + 8a + b \quad \lim_{x \to 2^+} f(x) = 12b - 12a + 12$

$$\text{so } 4 + 8a + b = 12b - 12a + 12 \rightarrow a = \frac{-63}{13} \quad b = \frac{-124}{13}$$

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(4) $\lim_{x \to 1^-} f(x) = 3 - 5a + 2b \quad \lim_{x \to 1^+} f(x) = -1 + 2a + b$

$$\text{so } 3 - 5a + 2b = -1 + 2a + b$$

$\lim_{x \to 2^-} f(x) = -4 + 4a + b \quad \lim_{x \to 2^+} f(x) = 4 - 6a + 3b$

$$\text{so } -4 + 4a + b = 4 - 6a + 3b$$

$$a = 0 \quad b = -4$$

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(5) $\lim_{x \to -1^-} f(x) = -a + 3b + 5 \quad \lim_{x \to -1^+} f(x) = 1 + 2a + 6b$

$$-a + 3b + 5 = 1 + 2a + 6b$$

$\lim_{x \to 1^-} f(x) = 1 - 2a + 6b \quad \lim_{x \to 1^+} f(x) = 1 + a + 2b$

$$\text{so } 1 - 2a + 6b = 1 + a + 2b$$

$$a = \frac{16}{21} \quad b = \frac{4}{7}$$

① secant Line: $\frac{f(4)-f(2)}{4-2} = \frac{26-14}{4-2} = 6$

tangent line: $\lim_{h \to 0} \frac{((x+h)^2+10)-(x^2+10)}{h} = \lim_{h \to 0} \frac{x^2+2xh+h^2+10-x^2-10}{h}$

$= \lim_{h \to 0} \frac{2xh+h^2}{h} \lim_{h \to 0} 2x+h = 2x$

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② secant line: $\frac{f(6)-f(2)}{6-2} = \frac{41-5}{6-2} = 9$

tangent line: $\lim_{h \to 0} \frac{(3(x+h)^2-7)-(3x^2-7)}{h} = \lim_{h \to 0} \frac{3(x^2+2xh+h^2)-7-3x^2+7}{h}$

$= \lim_{h \to 0} \frac{3x^2+6xh+3h^2-7-3x^2+7}{h} = \lim_{h \to 0} \frac{6xh+3h^2}{h}$

$= \lim_{h \to 0} 6x+3h = 6x$

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③ secant Line: $\frac{f(3)-f(1)}{3-1} = \frac{21-11}{3-1} = 5$

tangent line: $\lim_{h \to 0} \frac{((x+h)^2+(x+h)+9)-(x^2+x+9)}{h}$

$= \lim_{h \to 0} \frac{x^2+2xh+h^2+x+h+9-x^2-x-9}{h}$

$= \lim_{h \to 0} \frac{2xh+h^2+h}{h} = \lim_{h \to 0} 2x+h+1 = 2x+1$

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④ secant line: $\dfrac{f(4)-f(-2)}{4-(-2)} = \dfrac{40-(-16)}{6} = \dfrac{28}{3}$

tangent line: $\lim\limits_{h\to 0} \dfrac{[-(x+h)^2+6(x+h)]-(-x^2+6x)}{h}$

$= \lim\limits_{h\to 0} \dfrac{-(x^2+2xh+h^2)+6x+6h+x^2-6x}{h}$

$= \lim\limits_{h\to 0} \dfrac{-x^2-2xh-h^2+6x+6h+x^2-6x}{h}$

$= \lim\limits_{h\to 0} \dfrac{-2xh-h^2+6h}{h} = \lim\limits_{h\to 0} -2x-h+6 = -2x+6$

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⑤ secant line: $\dfrac{f(2)-f(1)}{2-1} = \dfrac{16-(-3)}{2-1} = 19$

tangent line: $\lim\limits_{h\to 0} \dfrac{[(x+h)^3+4(x+h)^2-8]-[x^3+4x^2-8]}{h}$

$= \lim\limits_{h\to 0} \dfrac{x^3+3x^2h+3xh^2+h^3+4x^2+8xh+4h^2-8-x^3-4x^2+8}{h}$

$= \lim\limits_{h\to 0} \dfrac{3x^2h+3xh^2+h^3+8xh+4h^2}{h}$

$= \lim\limits_{h\to 0} 3x^2+3xh+h^2+8x+4h = 3x^2+8x$

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⑥ secant line: $\dfrac{f(-1)-f(1)}{-1-1} = \dfrac{6-8}{-2} = 1$

tangent line: $\lim\limits_{h\to 0} \dfrac{\left(\frac{1}{x+h}+7\right)-\left(\frac{1}{x}+7\right)}{h} = \lim\limits_{h\to 0} \dfrac{\frac{1}{x+h}-\frac{1}{x}}{h}$

$= \lim\limits_{h\to 0} \dfrac{\frac{x-(x+h)}{x(x+h)}}{h} = \lim\limits_{h\to 0} \dfrac{-h}{x(x+h)h} = \lim\limits_{h\to 0} \dfrac{-1}{x(x+h)}$

$= \dfrac{-1}{x^2}$

7. secant line: $\dfrac{f(4)-f(1)}{4-1} = \dfrac{\frac{1}{13}-\frac{1}{7}}{4-1} = \dfrac{-2}{91}$

tangent line: $\lim_{h\to 0} \dfrac{\frac{1}{2(x+h)+5} - \frac{1}{2x+5}}{h} = \lim_{h\to 0} \dfrac{\frac{1}{2x+2h+5} - \frac{1}{2x+5}}{h}$

$= \lim_{h\to 0} \dfrac{\frac{(2x+5)-(2x+2h+5)}{(2x+2h+5)(2x+5)}}{h} = \lim_{h\to 0} \dfrac{-2h}{h(2x+2h+5)(2x+5)}$

$= \lim_{h\to 0} \dfrac{-2}{(2x+2h+5)(2x+5)} = \dfrac{-2}{(2x+5)^2}$

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8. secant line: $\dfrac{f(3)-f(2)}{3-2} = \dfrac{14-9}{3-2} = 5$

tangent line: $\lim_{h\to 0} \dfrac{\sqrt{x+h-5} - \sqrt{x-5}}{h} \cdot \dfrac{\sqrt{x+h-5}+\sqrt{x-5}}{\sqrt{x+h-5}+\sqrt{x-5}}$

$= \lim_{h\to 0} \dfrac{(x+h-5)-(x-5)}{h(\sqrt{x+h-5}+\sqrt{x-5})} = \lim_{h\to 0} \dfrac{h}{h(\sqrt{x+h-5}+\sqrt{x-5})}$

$= \lim_{h\to 0} \dfrac{1}{\sqrt{x+h-5}+\sqrt{x-5}} = \dfrac{1}{2\sqrt{x-5}}$

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9. secant line: $\dfrac{f(4)-f(1)}{4-1} = \dfrac{\frac{9}{4}-2}{4-1} = \dfrac{\frac{1}{4}}{3} = \dfrac{1}{12}$

tangent line: $\lim_{h\to 0} \dfrac{\left(\frac{1}{x+h}+\sqrt{x+h}\right)-\left(\frac{1}{x}+\sqrt{x}\right)}{h}$

$= \lim_{h\to 0} \dfrac{\frac{1}{x+h}-\frac{1}{x}}{h} + \dfrac{\sqrt{x+h}-\sqrt{x}}{h} = \lim_{h\to 0} \dfrac{\frac{x-(x+h)}{x(x+h)}}{h} + \dfrac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \dfrac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$

$= \lim_{h\to 0} \dfrac{-h}{hx(x+h)} + \dfrac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \dfrac{-1}{x(x+h)} + \dfrac{h}{h(\sqrt{x+h}+\sqrt{x})}$

$= \lim_{h\to 0} \dfrac{-1}{x(x+h)} + \dfrac{1}{\sqrt{x+h}+\sqrt{x}} = \dfrac{-1}{x^2} + \dfrac{1}{2\sqrt{x}}$

(10) secant line: $\frac{f(2)-f(1)}{2-1} = \frac{\frac{1}{4}-1}{2-1} = \frac{-3}{4}$

tangent line: $\lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \lim_{h \to 0} \frac{\frac{x^2-(x+h)^2}{x^2(x+h)^2}}{h}$

$= \lim_{h \to 0} \frac{x^2-(x^2+2xh+h^2)}{hx^2(x+h)^2} = \lim_{h \to 0} \frac{-2xh+h^2}{hx^2(x+h)^2}$

$= \lim_{h \to 0} \frac{-2x+h}{x^2(x+h)^2} = \frac{-2x}{x^4} = \frac{-2}{x^3}$