Solutions

Simple Derivatives

1) $y = 7x^2$ $\qquad \dfrac{dy}{dx} = 14x$

2) $y = -5x^{12}$ $\qquad \dfrac{dy}{dx} = -60x^{11}$

3) $y = 4x^{-2}$ $\qquad \dfrac{dy}{dx} = -8x^{-3}$

4) $y = x^{-1}$ $\qquad \dfrac{dy}{dx} = -x^{-2} = -\dfrac{1}{x^2}$

5) $y = \sqrt{x}$ $\qquad \dfrac{dy}{dx} = \dfrac{1}{2}x^{-\frac{1}{2}} = \dfrac{1}{2\sqrt{x}}$

6) $y = \sqrt[4]{x}$ $\qquad \dfrac{dy}{dx} = \dfrac{1}{4}x^{-\frac{3}{4}} = \dfrac{1}{4\sqrt[4]{x^3}}$

7) $y = \sqrt{x^5}$ $\qquad \dfrac{dy}{dx} = \dfrac{5}{2}x^{\frac{3}{2}} = \dfrac{5}{2\sqrt{x^3}}$

8) $y = \dfrac{15}{x^3}$ $\qquad \dfrac{dy}{dx} = -45x^{-4} = -\dfrac{45}{x^4}$

9) $y=\dfrac{9}{\sqrt{x}}$ $\qquad \dfrac{dy}{dx} = -\dfrac{9}{2}x^{-\frac{3}{2}} = -\dfrac{9}{2\sqrt{x^3}}$

10) $y=\sqrt[5]{x^3}$ $\frac{dy}{dx}=\frac{3}{5}x^{-\frac{2}{5}}=\frac{3}{5\sqrt[5]{x^2}}$

Derivatives

1) $y=x^3+x^2+1$ $\frac{dy}{dx}=3x^2+2x$

2) $y=6x^3-4x^2+2x$ $\frac{dy}{dx}=18x^2-8x+2$

3) $y=9x^{10}+10x^9$ $\frac{dy}{dx}=90x^9+90x^8$

4) $y=4x^4+6x^2+8+\frac{2}{x}$ $\frac{dy}{dx}=16x^3+12x-\frac{2}{x^2}$

5) $y=\sqrt{x}-\frac{1}{\sqrt{x}}$ $\frac{dy}{dx}=\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{x^3}}$

6) $y=\frac{3}{x}+\frac{3}{x^2}-\frac{3}{x^3}$ $\frac{dy}{dx}=-\frac{3}{x^2}-\frac{6}{x^3}+\frac{9}{x^4}$

7) $y=x^2+4x+10-\frac{2}{x}-\frac{6}{x^2}$ $\frac{dy}{dx}=2x+4+\frac{2}{x^2}+\frac{12}{x^3}$

8) $y=3\sqrt[5]{x}+15\sqrt{x}+\frac{3}{\sqrt[5]{x}}+\frac{15}{\sqrt{x}}$ $\frac{dy}{dx}=\frac{3}{5\sqrt[5]{x^4}}+\frac{15}{2\sqrt{x}}-\frac{3}{5\sqrt[5]{x^6}}-\frac{15}{\sqrt{x^3}}$

9) $y=\frac{x^{n+1}}{n+1}+\frac{x^n}{n}+\frac{x^{n-1}}{n-1}$ $\frac{dy}{dx}=x^n+x^{n-1}+x^{n-2}$

10) $y=x^{n+1}+x^n+x^{n-1}$ $\frac{dy}{dx}=(n+1)x^n+nx^{n-1}+(n-1)x^{n-2}$

Product Rule

1) $y=\left(x^2+2x+1\right)\left(x^2+3x+2\right)$

$$\frac{dy}{dx}=\left(x^2+2x+1\right)\left(2x+3\right)+\left(2x+2\right)\left(x^2+3x+2\right)$$

2) $y=\left(5x^2+x\right)\left(3x^3+8x+7\right)$

$$\frac{dy}{dx}=\left(5x^2+x\right)\left(9x^2+8\right)+\left(10x+1\right)\left(3x^3+8x+7\right)$$

3) $y=\left(4x^3-3x-2\right)\left(6x^2+7\right)$

$$\frac{dy}{dx}=\left(4x^3-3x-2\right)\left(12x\right)+\left(12x^2-3\right)\left(6x^2+7\right)$$

4) $y=x^2e^x$ $\qquad \frac{dy}{dx}=x^2e^x+2xe^x=e^x\left(x^2+2x\right)$

5) $y=x^2\sin x$ $\qquad \frac{dy}{dx}=x^2\cos x+2x\sin x$

6) $y=e^x\cos x$ $\qquad \frac{dy}{dx}=-e^x\sin x+e^x\cos x=e^x\left(\cos x-\sin x\right)$

7) $y=\sqrt{x}\tan x$ $\qquad \frac{dy}{dx}=\sqrt{x}\sec^2 x+\frac{\tan x}{2\sqrt{x}}$

8) $y=\sec x\tan x$ $\qquad \frac{dy}{dx}=\sec^3 x+\sec x\tan^2 x=\sec x\left(\sec^2 x+\tan^2 x\right)$

9) $y=e^x\csc x$ $\qquad \frac{dy}{dx}=-e^x\csc x\cot x+e^x\csc x=e^x\csc x\left(1-\cot x\right)$

10) $y = \cot x \tan x$ $\qquad \dfrac{dy}{dx} = 0$

Quotient Rule

1) $y = \dfrac{5x+1}{2-x}$ $\qquad \dfrac{dy}{dx} = \dfrac{(2-x)(5)-(5x+1)(-1)}{(2-x)^2} = \dfrac{11}{(2-x)^2}$

2) $y = \dfrac{3x^2+2x+4}{x^2-1}$

$$\frac{dy}{dx} = \frac{(x^2-1)(6x+2)-(3x^2+2x+4)(2x)}{(x^2-1)^2} = \frac{-2x^2-10x-2}{(x^2-1)^2}$$

3) $y = \dfrac{6x^2-4}{3x^2+4x-8}$

$$\frac{dy}{dx} = \frac{(3x^2+4x-8)(12x)-(6x^2-4)(6x+4)}{(3x^2+4x-8)^2} = \frac{72x^2-120x-16}{(3x^2+4x-8)^2}$$

4) $y = \dfrac{x^2}{e^x}$ $\qquad \dfrac{dy}{dx} = \dfrac{(e^x)(2x)-(x^2)(e^x)}{e^{2x}} = \dfrac{2x-x^2}{e^x}$

5) $y = \dfrac{\sin x}{e^x}$ $\qquad \dfrac{dy}{dx} = \dfrac{e^x\cos x - e^x \sin x}{e^{2x}} = \dfrac{\cos x - \sin x}{e^x}$

6) $$y = \frac{x + \frac{1}{x}}{x^2 - \frac{1}{x^2}}.$$

$$\frac{dy}{dx} = y = \frac{x + \frac{1}{x}}{x^2 - \frac{1}{x^2}} = \frac{\left(x^2 - \frac{1}{x^2}\right)\left(1 - \frac{1}{x^2}\right) - \left(x + \frac{1}{x}\right)\left(2x + \frac{2}{x^3}\right)}{\left(x^2 - \frac{1}{x^2}\right)^2}$$

7) $$y = \frac{\sqrt{x} + \tan x}{\sqrt{x} - \cot x}$$

$$\frac{dy}{dx} = y = \frac{\left(\sqrt{x} - \cot x\right)\left(\frac{1}{2\sqrt{x}} + \sec^2 x\right) - \left(\sqrt{x} + \tan x\right)\left(\frac{1}{2\sqrt{x}} + \csc^2 x\right)}{\sqrt{x} - \cot x}$$

8) $$y = \frac{\sec x}{\csc x} \qquad \frac{dy}{dx} = \sec^2 x$$

9) $$y = \frac{e^x + 1}{e^x - 1} \qquad \frac{dy}{dx} = \frac{\left(e^x - 1\right)e^x - \left(e^x + 1\right)e^x}{\left(e^x - 1\right)^2} = -\frac{2e^x}{\left(e^x - 1\right)^2}$$

10) $$y = \frac{\tan x}{\cot x} \qquad \frac{dy}{dx} = \frac{\cot x \sec^2 x - \tan x \csc^2 x}{\cot^2 x} = \frac{2\sin x}{\cos^3 x}$$

Chain Rule

Find the derivative of each of the following:

1) $$y = \left(x^2 + 4x + 1\right)^3 \qquad \frac{dy}{dx} = 3\left(x^2 + 4x + 1\right)^2\left(2x + 4\right)$$

2) $$y = \left(8x^3 + 7\right)^6 \qquad \frac{dy}{dx} = 6\left(8x^3 + 7\right)^5\left(24x^2\right)$$

3) $y=\left(10x^8+6x+1\right)^4$ $\qquad$ $\dfrac{dy}{dx}=4\left(10x^8+6x+1\right)^3\left(80x^7+6\right)$

4) $y=\sqrt{9-3x}$ $\qquad$ $\dfrac{dy}{dx}=\dfrac{1}{2}(9-3x)^{-\frac{1}{2}}(-3)=-\dfrac{3}{2\sqrt{9-3x}}$

5) $y=e^{\frac{x}{4}}$ $\qquad$ $\dfrac{dy}{dx}=\dfrac{1}{4}e^{\frac{x}{4}}$

6) $y=(\sin x+\cos x)^{\frac{2}{3}}$ $\qquad$ $\dfrac{dy}{dx}=\dfrac{2}{3}(\sin x+\cos x)^{-\frac{1}{3}}(\cos x-\sin x)$

7) $y=e^{x^2}\sin 2x$

$$\frac{dy}{dx}=e^{x^2}\left(2\cos 2x\right)+2xe^{x^2}\sin 2x=2e^{x^2}\left(\cos 2x+x\sin 2x\right)$$

8) $y=\sec^4 \pi x$ $\qquad$ $\dfrac{dy}{dx}=4\pi\sec^4 \pi x\tan \pi x$

9) $y=\tan^2(\sin x)$ $\qquad$ $\dfrac{dy}{dx}=2\tan(\sin x)\sec^2(\sin x)\cos x$

10) $y=(\sin 2x+\cos 2x)^2$

$$\frac{dy}{dx}=2(\sin 2x+\cos 2x)(2\cos 2x-2\sin 2x)=4\cos 4x$$