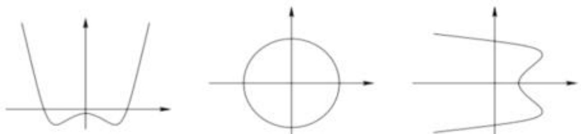
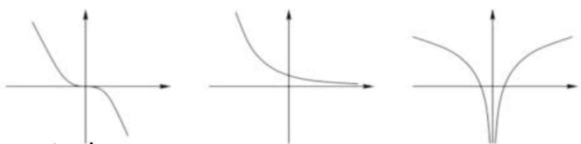
1. Specify whether the graph of the function in each part is odd, even or neither:



even not a func. not a func.



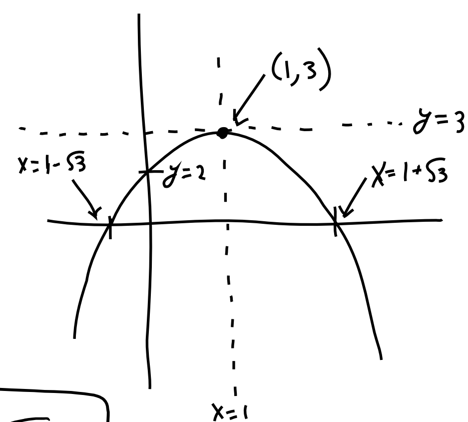
odd neither even

2. For the parabola $y = -x^2 + 2x + 2$, find the coordinates of the vertex, an equation of the axis of symmetry, and the x and y intercepts. Draw the graph. Label your picture properly: indicate the vertex, the axis of symmetry, and the x and y intercepts.

$$y = -x^2 + 2x + 2$$
$$= -(x^2 - 2x - 2)$$
$$= -([x^2 - 2x + (1)] + [(-1) - 2])$$
$$= -((x-1)^2 - 3) = -(x-1)^2 + 3$$

(completing the square)

$\Longrightarrow$ Concave down Parabola
- Shifted 1 unit to the right
- Shifted 3 units up



Vertex: $(1, 3)$

Axis of symm: $x = 1$

x-int: $-(x-1)^2 + 3 = 0$
$\Longrightarrow (x-1)^2 = 3 \Longrightarrow \boxed{x = 1 \pm \sqrt{3}}$

y-int: $\boxed{y = 2}$

3. Let $f(x) = 5^x, g(x) = 2x + 3$. Find $f \circ g, g \circ f$, and $g \circ g$.

$(f \circ g)(x) = f(g(x)) = f(2x+3) = 5^{2x+3}$

$(g \circ f)(x) = g(f(x)) = g(5^x) = 2(5^x) + 3$

$(g \circ g)(x) = g(g(x)) = 2(2x+3) + 3 = 4x + 9$

4. Find the domain and range for each of the following functions. Also, write each as a composition of two functions or three functions where possible (do not choose the inner most function to x. We're not looking for triviality):

(a) $f(x) = |x + 1|$

(b) $y = 3^{x+1}$

(c) $f(t) = \sin(\ln(t - 3))$

(d) $g(u) = (u + 1)^{\frac{1}{4}}$

(a) $G(x) = x+1$, $F(x) = |x|$, dom f: $x \in \mathbb{R}$, ran g: $y \in \mathbb{R}^+$

(b) $G(x) = x+1$, $F(x) = 3^x$, dom f: $x \in \mathbb{R}$, ran f: $y \in \mathbb{R}^+$

(c) As two funct: $G(x) = \ln(t-3)$
$F(x) = \sin(t)$

dom f: $t > 3$
ran f: $y \in [-1,1]$

As three funct: $H(t) = t - 3$
$G(t) = \ln(t)$
$F(t) = \sin(t)$

(d) $G(u) = u+1$
$F(u) = u^{1/4}$

dom g: $u \in [-1, \infty)$
ran g: $u \in \mathbb{R}^+$

5. Simplify the following expressions:

(a) $\log_3(\sqrt{27})$

(b) $2^{\log_{\frac{1}{2}}(\sqrt[3]{64})}$

(a) $\log_3(\sqrt{27}) = \log_3(27^{\frac{1}{2}})$
$= \frac{1}{2}\log_3(27) = \frac{1}{2}\log_3(3^3)$

?

$$= \frac{3}{2} \log_3(3) = \frac{3}{2}$$

(b) $2^{\log_{\frac{1}{2}}(\sqrt[3]{64})}$ (1)

Zone in: $\log_{\frac{1}{2}}(\sqrt[3]{64}) = \log_{\frac{1}{2}}(64^{1/3})$ (2)

$$= \frac{1}{3}\log_{\frac{1}{2}}(64)$$

Zone in: $\log_{\frac{1}{2}}(64) = X$ (3)

$$\Longleftrightarrow \left(\frac{1}{2}\right)^X = 64$$

$$\Longrightarrow X = -6$$

Back to (2)

$$\Longrightarrow \frac{1}{3}\log_{\frac{1}{2}}(64) = \frac{-6}{3} = -2$$

Back to (1)

$$\Longrightarrow \boxed{2^{\log_{\frac{1}{2}}(\sqrt[3]{64})} = 2^{-2} = \frac{1}{4}}$$

6. In each of the following cases, find the domain of the given function, write it as a composition of two or three functions, and say whether the function is even, odd, or neither:

   (a) $x + \frac{1}{x}$

   (b) $\frac{x^3 - x}{x^3 + x}$

   (c) $|x|$

   (d) $\frac{x}{|x|}$

   (e) $\sqrt{x^4 + x^2 + 1}$

Let each function be $f(x)$. Then

(a) dom f: $\{X \neq 0\} \equiv (-\infty, 0) \cup (0, \infty)$

ran f: $y \in (-\infty, 0) \cup (0, \infty)$

$$f(x) = X + \frac{1}{X} = \frac{X^2+1}{X}$$

even · odd = odd

$f(x) = x + \frac{1}{x} = \frac{x^2+1}{x}$

$G(x) = x^2$

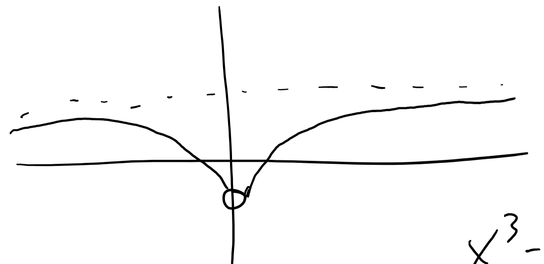
$F(x) = \frac{x+1}{\sqrt{x}}$

even · odd = odd

(b) dom f : $\{x \neq 0\} \equiv (-\infty, 0) \cup (0, \infty)$

ran f : $y \in (-1, 1)$

even func.



R.M.D.

$f(x) = \frac{x^3 - x}{x^3 + x} = \frac{\cancel{x}(x^2-1)}{\cancel{x}(x^2+1)} = \frac{x^2-1}{x^2+1}$

$G(x) = x^3$

$F(x) = \frac{x - \sqrt[3]{x}}{x + \sqrt[3]{x}}$

(c) dom f : $x \in \mathbb{R}$

ran f : $y \in \mathbb{R}^+$

even func.

Composition here is trivial

(d) dom f : $(-\infty, 0) \cup (0, \infty)$

ran f : $y = \{-1, 1\}$

odd func.

Composition here is trivial

(e) $\sqrt{x^4 + x^2 + 1}$

dom f : $x \in \mathbb{R}$

ran f : $y \in [1, \infty)$

neither odd nor even

$G(x) = x^4 + x^2 + 1$

$F(x) = \sqrt{x}$

7. Simplify the following:

(a) $27^{\frac{1}{3}}$

(b) $1 + x^{\frac{1}{3}} + x^{\frac{2}{3}}$

(c) $x^{\frac{1}{3}}x^{-\frac{1}{2}}$

(d) $\frac{x^2y^3}{(x^{-3}y^2)^{-3}}$

(e) $(\frac{81x^5}{125y^3})^{\frac{1}{3}}$

(a) $27^{1/3} = (3^3)^{1/3} = 3^{3/3} = \boxed{3}$

(b) Let $v = x^{1/3}$. Then have

$$v^2 + v + 1 = v^2 + v + \frac{1}{4} - \frac{1}{4} + 1$$ (Comp. the square)

$$= \left(v + \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$= \boxed{\left(x^{1/3} + \frac{1}{2}\right)^2 + \frac{3}{4}}$$ (since $v = x^{1/3}$)

(c) $x^{\frac{1}{3}}x^{-1/2} = x^{\frac{1}{3}-\frac{1}{2}} = x^{\frac{2-3}{6}} = x^{-\frac{1}{6}} = \boxed{\frac{1}{\sqrt[6]{x}}}$

(d) $\frac{x^2y^3}{(x^{-3}y^2)^{-3}} = \frac{x^2y^3}{(x^{-3})^{-3}(y^2)^{-3}} = \frac{x^2y^3}{x^9y^{-6}} = x^{2-9}y^{3-(-6)}$

$$= x^{-7}y^9 = \boxed{\frac{y^9}{x^7}}$$

(e) $\left(\frac{81x^5}{125y^3}\right)^{1/3} = \frac{(81x^5)^{1/3}}{(125y^3)^{1/3}} = \frac{(81)^{1/3}(x^5)^{1/3}}{(125)^{1/3}(y^3)^{1/3}}$

$$= \frac{(3^3\cdot 3)^{1/3}x^{\frac{5}{3}}}{(5^3)^{1/3}y^{\frac{3}{3}}} = \frac{(3^3)^{1/3}(3)^{1/3}x^{\frac{5}{3}}}{5y}$$

$$= \frac{3^{\frac{3}{3}}\sqrt[3]{3}\,x^{\frac{5}{3}}}{5y} = \boxed{\frac{\sqrt[3]{3}\cdot 3}{5}\cdot\frac{x^{\frac{5}{3}}}{y}}$$

8. Solve each of the following:

(a) $\log_5(x-1) = 2$

(b) $\log_2(8x) = 5$

(c) $\frac{\log_2(x)}{\log_2(3)} = 2$

(d) $\log_2(x+1) - \log_2(x-1) = 2$

(a) $\log_5(X-1)=2 \iff 5^{\log_5(X-1)} = 5^2$

$\overset{\text{inverse}}{\Longrightarrow} X-1 = 25$

$\Longrightarrow \boxed{X=26}$

(b) $\log_2(8X)=5 \iff 2^{\log_2(8X)} = 2^5$

$\overset{\text{inverse}}{\Longrightarrow} 8X=32 \Longrightarrow \boxed{X=4}$

(c) $\frac{\log_2(X)}{\log_2(3)}=2 \Longrightarrow \log_2(X) = 2\log_2(3)$

$\Longrightarrow \log_2(X) = \log_2(3^2)$

$\Longrightarrow 2^{\log_2(X)} = 2^{\log_2(9)}$

$\Longrightarrow \boxed{X=9}$

(d) $\log_2(X+1) - \log_2(X-1) = 2$

$\Longrightarrow \log_2\left(\frac{X+1}{X-1}\right) = 2$

$\Longrightarrow 2^{\log_2\left(\frac{X+1}{X-1}\right)} = 2^2$

$\Longrightarrow \frac{X+1}{X-1} = 4$

$\Longrightarrow X+1=4(X-1) \Longrightarrow X+1=4X-4$

$\Longrightarrow 3X=5$

$\Longrightarrow \boxed{X=\frac{5}{3}}$

9. In each of the following cases, find the center of the given ellipse:

(a) $4x^2 + 8x + y^2 - 2y = 11$

9. In each of the following cases, find the center of the given ellipse:

(a) $4x^2 + 8x + y^2 - 2y = 11$

(b) $x^2 + 2x + 4y^2 + 24y = -36$

(c) $9x^2 + 36x + y^2 - 10y + 60 = 0$

(d) $9x^2 - 54x + 4y^2 + 8y + 49$ $= 0$

I use completing the square for everything:

(a) $4(x^2+2x) + (y^2-2y) = 11$

$\Rightarrow 4((x+1)^2 - 1) + [(y-1)^2 - 1] = 11$

$\Rightarrow 4(x+1)^2 + (y-1)^2 = 11 + 5 = 16$

$\Rightarrow 4(x+1)^2 + (y-1)^2 = 16$

$\Rightarrow$ $\boxed{\text{Center @ } (-1,1)}$

(b) $[x^2+2x] + 4[y^2+6y] = -36$

$\Rightarrow [(x+1)^2 - 1] + 4[(y+3)^2 - 3^2] = -36$

$\Rightarrow [(x+1)^2 - 1] + [4(y+3)^2 - 36] = -36$

$\Rightarrow (x+1)^2 + 4(y+3)^2 = 1$

$\Rightarrow$ Centered @ $(-1,-3)$

(c) $9(x^2+4x) + (y^2-10y) = -60$

$\Rightarrow 9[(x+2)^2 - 4] + [(y-5)^2 - 5^2] = -60$

$\Rightarrow 9(x+2)^2 - 36 + (y-5)^2 - 25 = -60$

$\Rightarrow 9(x+2)^2 + (y-5)^2 = 1$

$\Rightarrow$ Centered @ $(-2,5)$

(d) $9x^2 - 54x + 4y^2 + 8y + 49 = 0$

$\Rightarrow 9[(x-3)^2 - 9] + 4[(y+1)^2 - 1] = -49$

$\Rightarrow 9(x-3)^2 - 81 + 4(y+1)^2 - 4 = -49$

$\Rightarrow 9(x-3)^2 + 4(y+1)^2 = -49 + 85$

$\Rightarrow 9(x-3)^2 + 4(y+1)^2 = 36$

$\Rightarrow$ Center @ $(3,-1)$

10. In each of the following cases, find the following information:

(a) Zeroes of f.

(b) y-intercept.

(c) Sign of the function between the zeroes.

(d) The behavior of f as $x \to \infty$.

(e) Whether the function is odd, even, or neither.

(f) Give a rough sketch of the graph illustraing all of these features.

i. $f(t) = t(t^2 - 1)$

ii. $g(x) = x^3 - 9x$

iii. $h(u) = u^4 - 1$

iv. $j(x) = x^4 - 5x^2 + 4$

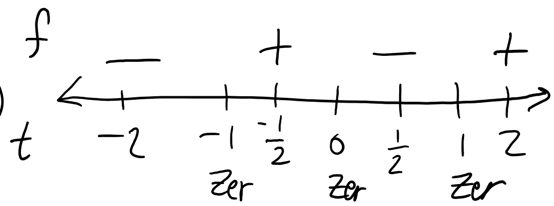
v. $k(n) = n^4 - 5n^3 + 4n^2$

(i) $y = t(t^2-1)$

(a) $t=0,\ t=\pm 1$

(b) $y=0$

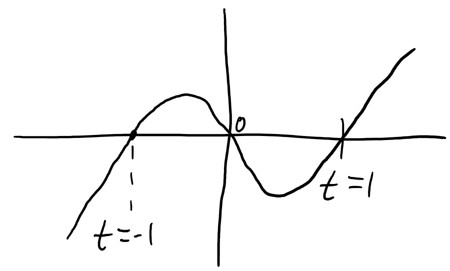
(c)



(d) As $t \to \infty,\ y \to \infty$

(e) odd · even = odd

(f)

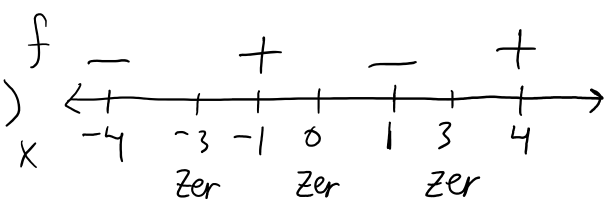


(ii) $y = x^3 - 9x = x(x^2-9)$

(a) $x=0,\ x=\pm 3$

(b) $y=0$

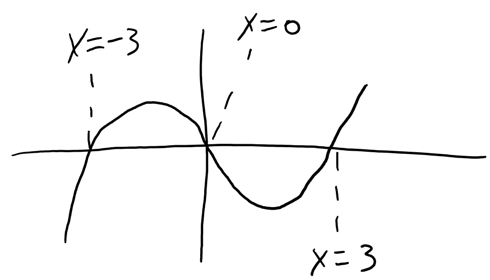
(c)



(d) As $x \to \infty,\ y \to \infty$

(e) odd

(f)

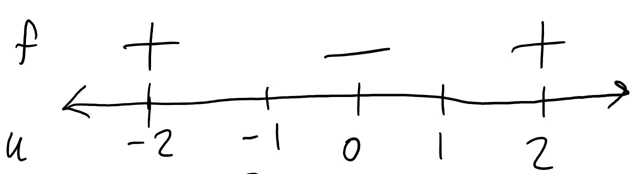


(iii) $y = u^4 - 1 = (u^2-1)(u^2+1)$

(a) $u = \pm 1$

(b) $y = -1$

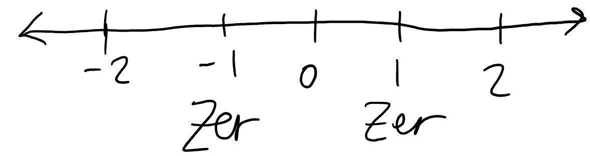
(c)



(a) $u = \pm 1$

(b) $y = -1$

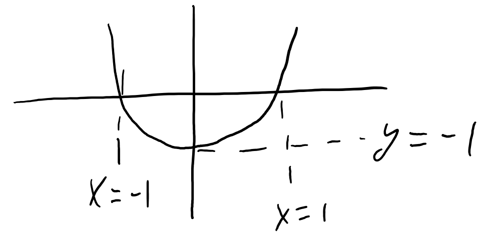
(c)



(d) As $u \to \infty$, $y \to \infty$

(e) even

(f)

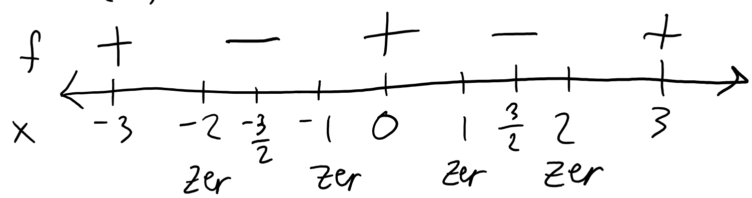


(iv) $y = x^4 - 5x^2 + 4 = (x^2-4)(x^2-1) = (x-2)(x+2)(x-1)(x+1)$

(a) $x = \pm 2$, $x = \pm 1$

(b) $y = 4$

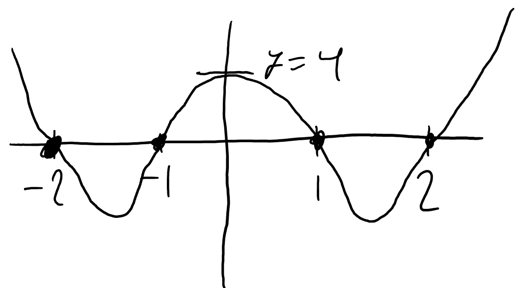
(c)



(d) As $x \to \infty$, $y \to \infty$

(e) even

(f)

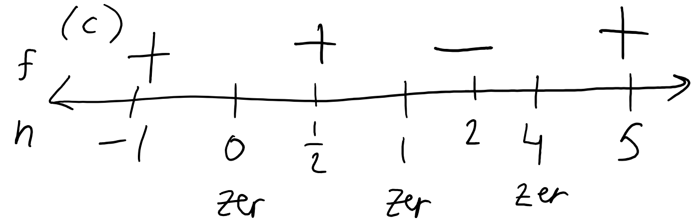


(v) $y = h^4 - 5h^3 + 4h^2 = h^2(h^2 - 5h + 4) = h^2(h-4)(h-1)$

(a) $h = 0, h = 1, h = 4$

(b) $y = 0$

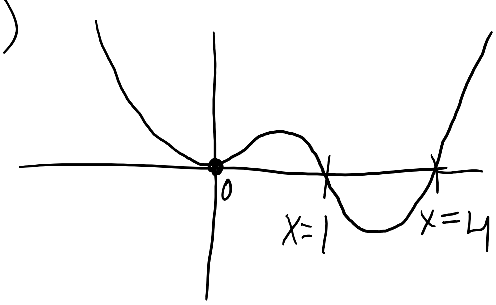
(c)



(d) As $h \to \infty$, $y \to \infty$

(e) even

(f)



11. Justify/Prove the following:

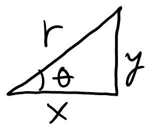
(a) $\cos^2(\theta) + \sin^2(\theta) = 1$.

(b) $\cos(-\theta) = \cos(\theta)$

(c) $\sin(-\theta) = -\sin(\theta)$

(d) $\tan(-\theta) = \tan(\theta)$

(a)



$\Rightarrow \sin\theta = \frac{O}{H} = \frac{y}{r} \Rightarrow y = r\sin\theta$

$\cos\theta = \frac{A}{H} = \frac{x}{r} \Rightarrow x = r\cos\theta$

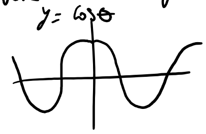
By Pythag Thm,

$x^2 + y^2 = r^2 \Rightarrow (r\cos\theta)^2 + (r\sin\theta)^2 = r^2$

$\Rightarrow \cancel{r^2}(\cos^2\theta + \sin^2\theta) = \cancel{r^2}$

$= \cos^2\theta + \sin^2\theta = 1$ ✓

(b) Look at the graph:

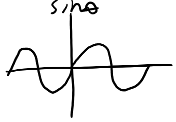


$\Rightarrow$ even func.

$\Rightarrow f(-\theta) = f(\theta)$ by def ✓

$\Rightarrow \cos(-\theta) = \cos(\theta)$

(c) Look at the graph:

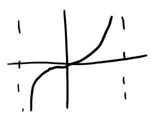


$\Rightarrow$ odd function

$\Rightarrow f(-\theta) = -f(\theta)$ by def

$\Rightarrow \sin(-\theta) = -\sin(\theta)$ ✓

(d) Look at the graph



$\Rightarrow$ odd function

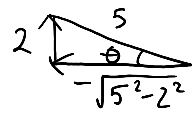
$\Rightarrow f(-\theta) = -f(\theta)$ by def ✓

$\Rightarrow \tan(-\theta) = -\tan(\theta)$

12. Let θ be an angle such that $\frac{\pi}{2} < \theta < \pi$ and $\sin(\theta) = \frac{2}{5}$. Find $cos(\theta)$.

2^{nd} quadrant $\Rightarrow$ $\cos\theta$ produces neg. quantity

$\sin\theta = \frac{2}{5} = \frac{O}{H} \Rightarrow$



$\Rightarrow A = -\sqrt{21}$

$\Rightarrow \boxed{\cos\theta = -\frac{\sqrt{21}}{5}}$

13. In each of the following cases, convert the given degree measure of an angle to the corresponding radian measure of an angle:

(a) $30°$

(b) $75°$

(c) $-120°$

(d) $200°$

(e) $(\frac{200}{\pi})°$

(f) $285°$

(g) $-780°$

(h) $135°$

(a) $30° = 30 \cdot \frac{\pi}{180} = \boxed{\frac{\pi}{6}}$ rad

(b) $75° = 75 \cdot \frac{\pi}{180} = \frac{(5 \cdot \cancel{15})\pi}{(\cancel{15} \cdot 12)} = \boxed{\frac{5\pi}{12}}$ rad

(c) $-120° = -120 \cdot \frac{\pi}{180} = \boxed{-\frac{2\pi}{3}}$ rad

(d) $200° = \frac{20\cancel{0} \cdot \pi}{18\cancel{0}} = \boxed{\frac{10\pi}{9}}$ rad

(e) $(\frac{200}{\pi})° = \frac{200 \cdot \cancel{\pi}}{180 \cancel{\pi}} = \boxed{\frac{10}{9}}$ rad

(f) $285° = \frac{285}{180}\pi = \frac{57 \cdot \cancel{5}}{36 \cdot \cancel{5}}\pi = \frac{(\cancel{3} \cdot 19)}{(\cancel{3} \cdot 12)}\pi = \boxed{\frac{19}{12}\pi}$ rad

(g) $-780° = \frac{-78\cancel{0} \cdot \pi}{18\cancel{0}} = \frac{-39}{9}\pi = \boxed{\frac{-13}{3}\pi}$

(h) $135° = \frac{135\pi}{180} = \boxed{\frac{3\pi}{4}}$ rad

14. In each case, the cosine of 2θ is given and an interval of θ is given. Find a quadratic equation satisfied by $cos\theta$ and $sin\theta$, and then solve the equation.

(a) $\cos(2\theta) = \frac{\sqrt{2}}{2}$, $\theta \in [0, \frac{\pi}{2})$

(b) $\cos(2\theta) = \frac{3}{4}$, $\theta \in [-\frac{\pi}{2}, 0)$

(c) $\cos(2\theta) = \frac{1}{2}$, $\theta \in (0, \frac{\pi}{2}]$

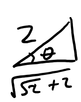
(d) $\cos(2\theta) = \sqrt{2 + \frac{\sqrt{2}}{2}}$, $\theta \in [0, \frac{\pi}{2}]$

(e) $\cos(2\theta) = 1$, $\theta \in (\frac{\pi}{2}, \pi]$

(a) $\cos(2\theta) = \frac{\sqrt{2}}{2} = 2\cos^2\theta - 1$

1st quad

$= \frac{\sqrt{2}+2}{4} = \cos^2\theta \Rightarrow \boxed{\cos\theta = \frac{\sqrt{\sqrt{2}+2}}{2}}$

$\Rightarrow$ 

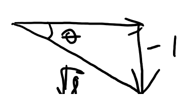
$4 = \sqrt{2} + 2 + b^2$

$2 - \sqrt{2} = b^2 \Rightarrow b = \sqrt{2-\sqrt{2}}$

$\Rightarrow \boxed{\sin\theta = \frac{\sqrt{2-\sqrt{2}}}{2}}$

(b) $\cos(2\theta) = \frac{3}{4} = 1 - 2\sin^2\theta$

4th quad

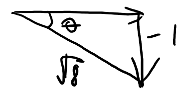
$\Rightarrow 2\sin^2\theta = \frac{1}{4} \Rightarrow$ 

(b) $\cos(2\theta) = \frac{3}{4} = 1 - 2\sin^2\theta$

4th quad

$\Rightarrow 2\sin^2\theta = \frac{1}{4} \Rightarrow$

$\Rightarrow \boxed{\sin\theta = -\frac{1}{\sqrt{8}} \qquad \cos\theta = \frac{\sqrt{7}}{\sqrt{8}}}$



$8 = 1 + b^2$

$\Rightarrow b = \sqrt{7}$

(c) $\cos(2\theta) = \frac{1}{2} = 2\cos^2\theta - 1$

1st quad

$\Rightarrow \boxed{\cos\theta = \frac{\sqrt{3}}{2}}$

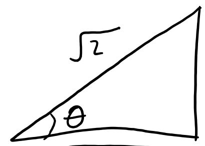
$\Rightarrow \boxed{\sin\theta = \frac{1}{2}}$

(d) $\cos(2\theta) = \sqrt{2 - \frac{\sqrt{2}}{2}} = 2\cos^2\theta - 1$

1st quad

$\Rightarrow \cos^2\theta = \frac{\sqrt{2 - \frac{\sqrt{2}}{2}} + 1}{2}$

$\Rightarrow \boxed{\cos\theta = \sqrt{\frac{\sqrt{2 - \frac{\sqrt{2}}{2}} + 1}{2}} \qquad \sin\theta = \sqrt{\frac{1 - \sqrt{2 - \frac{\sqrt{2}}{2}}}{2}}}$



$\sqrt{\sqrt{2 - \frac{\sqrt{2}}{2}} + 1}$

$2 = \sqrt{2 - \frac{\sqrt{2}}{2}} + 1 + b^2$

$1 - \sqrt{2 - \frac{\sqrt{2}}{2}} = b^2$

$\Rightarrow b = \sqrt{1 - \sqrt{2 - \frac{\sqrt{2}}{2}}}$

(e) $\cos(2\theta) = 1 = 1 - 2\sin^2\theta$

2nd quad

$\Rightarrow 2\sin^2\theta = 0$

$\Rightarrow \boxed{\sin\theta = 0 \qquad \cos\theta = -1}$ $(\theta = \pi)$

15. Verify each statement:

(a) $\cos(3\theta) = 4\cos^3\theta - 3\cos\theta$

(b) $\sin(3\theta) == 4\sin^3\theta + 3\sin\theta$

(c) $\tan(2\theta) = \frac{2\tan\theta}{\cancel{1+\tan^2\theta}}$ $1 - \tan^2\theta$

(d) $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

(e) $\sec(2\theta) = \frac{\sec^2\theta}{2 - \sec^2\theta}$

(f) $\csc(2\theta) = \frac{1}{2}\sec\theta\csc\theta$

(g) $\frac{\sin(2\theta)}{1+\cos(2\theta)} = \frac{1-\cos(2\theta)}{\sin(2\theta)}$

(h) $\sin(2\theta) = \frac{2\tan\theta}{1+\tan^2\theta}$

(i) $\cos(2\theta) = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

(a) $\cos(3\theta)$

$= \cos(2\theta + \theta)$

$= \cos(2\theta)\cos(\theta) - \sin(2\theta)\sin\theta$

$= [2\cos^2\theta - 1]\cos\theta - 2\sin^2\theta\cos\theta$

$= \cos\theta[2(\cos^2\theta - \sin^2\theta) - 1]$

$= \cos\theta[2[2\cos^2\theta - 1] - 1]$

$= 4\cos^3\theta - 3\cos\theta$ ✓

(b) $\sin(3\theta) = \sin(2\theta + \theta) = \sin(2\theta)\cos(\theta) + \cos(2\theta)\sin\theta$

$= 2\sin\theta\cos^2\theta + [2\cos^2\theta - 1]\sin\theta$

$= \sin\theta[4\cos^2\theta - 1]$

$$= \sin\theta\left[4\cos^2\theta - 1\right]$$

$$= \sin\theta\left[4 - 4\sin^2\theta - 1\right] = 3\sin\theta - 4\sin^3\theta \quad ✓$$

(c) $$\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)} = \frac{2\sin\theta\cos\theta}{2\cos^2\theta - 1} = \frac{2\sin\theta\cos\theta}{\cos^2\theta(2 - \sec^2\theta)}$$

$$= \frac{2\tan\theta}{1 - \tan^2\theta} \quad ✓$$

(d) $$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta - \sin\alpha\sin\beta}$$

$$= \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta\left(1 - \frac{\sin\alpha}{\cos\alpha}\cdot\frac{\sin\beta}{\cos\beta}\right)} = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \quad ✓$$

(e) $$\sec(2\theta) = \frac{1}{\cos(2\theta)} = \frac{1}{2\cos^2\theta - 1} = \frac{1}{\cos^2\theta(2 - \sec^2\theta)} = \frac{\sec^2\theta}{2 - \sec^2\theta} \quad ✓$$

(f) $$\csc(2\theta) = \frac{1}{\sin(2\theta)} = \frac{1}{2\sin\theta\cos\theta} = \frac{1}{2}\csc\theta\sec\theta$$

(g) $$\frac{\sin(2\theta)}{1+\cos(2\theta)} = \frac{\cancel{2}\sin\theta\cos\theta}{\cancel{1} + \cancel{2}\cos^2\theta \cancel{-1}} = \frac{\sin\theta}{\cos\theta} = \tan\theta \quad (1)$$

$$\frac{1-\cos(2\theta)}{\sin(2\theta)} = \frac{1 - (1 - 2\sin^2\theta)}{2\sin\theta\cos\theta}$$

$$= \frac{\cancel{2}\sin^2\theta}{\cancel{2}\sin\theta\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta \quad (2)$$

$$(1) = (2) \implies \frac{\sin(2\theta)}{1+\cos(2\theta)} = \frac{1-\cos(2\theta)}{\sin(2\theta)} \quad ✓$$

(h) $$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$= 2\frac{\sin\theta}{\cos\theta}\cos^2\theta = \frac{2\tan\theta}{\sec^2\theta} = \frac{2\tan\theta}{1+\tan^2\theta} \quad ✓$$

(i) $$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$= \cos^2\theta(1 - \tan^2\theta) = \frac{1-\tan^2\theta}{\sec^2\theta} = \frac{1-\tan^2\theta}{1+\tan^2\theta} \quad ✓$$

16. Compute each of the following values (no calculator work):

(a) $\sin^{-1}(\frac{1}{2})$

(b) $\cos^{-1}(\frac{1}{2})$

(c) $\tan^{-1}(\sqrt{3})$

(d) $\sin^{-1}(0)$

(e) $\cos^{-1}(0)$

(f) $\tan^{-1}(1)$

(g) $\sin^{-1}(\frac{-1}{\sqrt{2}})$

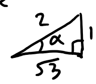
(h) $\cos^{-1}(\frac{-1}{\sqrt{2}})$

(i) $\tan^{-1}(0)$

(j) $\sin^{-1}(\frac{-\sqrt{3}}{2})$

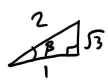
(k) $\cos^{-1}(\frac{-\sqrt{3}}{2})$

(a) $\sin^{-1}(\frac{1}{2}) = \alpha$

$\Rightarrow$ 

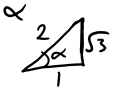
$\Rightarrow \boxed{\alpha = \frac{\pi}{6} = \sin^{-1}(1/2)}$

(b) $\cos^{-1}(\frac{1}{2}) = \beta$

$\Rightarrow$ 

$\Rightarrow \boxed{\beta = \frac{\pi}{3} = \cos^{-1}(1/2)}$

(c) $\tan^{-1}(\sqrt{3}) = \alpha$

$\Rightarrow$ 

$\Rightarrow \boxed{\alpha = \frac{\pi}{3} = \tan^{-1}(\sqrt{3})}$

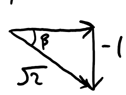
(d) $\boxed{\sin^{-1}(0) = 0}$, (e) $\boxed{\cos^{-1}(0) = \frac{\pi}{2}}$

(f) $\tan^{-1}(1) = \alpha$

$\Rightarrow$ 

$\Rightarrow \boxed{\tan^{-1}(1) = \frac{\pi}{4}}$

(g) $\sin^{-1}(-\frac{1}{\sqrt{2}}) = \beta$

$\Rightarrow$ 

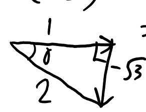
$\Rightarrow \boxed{\sin^{-1}(-\frac{1}{\sqrt{2}}) = -\frac{\pi}{4}}$

(h) $\cos^{-1}(-\frac{1}{\sqrt{2}}) = \theta$

$\Rightarrow$ 

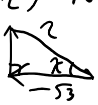
$\Rightarrow \boxed{\cos^{-1}(-\frac{1}{\sqrt{2}}) = \frac{3\pi}{4}}$

(i) $\boxed{\tan^{-1}(0) = 0}$, (j) $\sin^{-1}(-\frac{\sqrt{3}}{2}) = \gamma$

$\Rightarrow$ 

$\Rightarrow \boxed{\sin^{-1}(-\frac{\sqrt{3}}{2}) = -\frac{\pi}{3}}$

(k) $\cos^{-1}(-\frac{\sqrt{3}}{2}) = x$

$\Rightarrow$ 

$\Rightarrow \boxed{\cos^{-1}(-\frac{\sqrt{3}}{2}) = \frac{5\pi}{6}}$

17. Simplify the following using methods such as polynomial long division, synthetic division, or another method you think of:

(a) $\frac{2x^2+3x+1}{x}$

(b) $\frac{2x^2+3x+1}{x+1}$

(c) $\frac{x^2+x+1}{x^2-x+1}$

(d) $\frac{x^3+x^2+x+1}{x^2+x+1}$

(e) $\frac{x^2+2x+3}{3x-2}$

(a) $\boxed{2x+3+\frac{1}{x}}$

(b) Synthetic division:

$$\begin{array}{r|rrr} -1 & 2 & 3 & 1 \\ & \downarrow & -2 & -1 \\ \hline & 2 & 1 & 0 \end{array}$$

$\Rightarrow \boxed{2x+1}$ no remainder

(c) $\frac{x^2+x+1}{x^2-x+1} = \frac{x^2+(2x-x)+1}{x^2-x+1} = \frac{(x^2-x+1)+2x}{x^2-x+1} = \boxed{1+\frac{2x}{x^2-x+1}}$

(d) $\frac{x^3+x^2+x+1}{x^2+x+1} = \frac{x(x^2+x+1)+1}{x^2+x+1} = \boxed{x+\frac{1}{x^2+x+1}}$

(e) $\frac{x^2+2x+3}{3x-2}$; Synthetic Division

$3x=2 \Rightarrow x=\frac{2}{3}$

$$\begin{array}{r|rrr} \frac{2}{3} & 1 & 2 & 3 \\ & \downarrow & \frac{2}{3} & \frac{16}{9} \\ \hline & 1 & \frac{8}{3} & \frac{43}{9} \end{array}$$

$\Rightarrow \frac{x^2+2x+3}{3x-2} = \boxed{x+\frac{8}{3}+\frac{(43/9)}{3x-2}}$

18. Let $f(x) = e^{\sin(3x)}$.

(a) Present f as a composition of two functions. (Do not choose x as one of your functions)

(b) Present f as a composition of three functions. (Do not choose x as one of your functions)

(a) $G(x) = \sin(3x)$
$F(x) = e^x$

(b) $H(x) = 3x$
$G(x) = \sin x$

$F(X) = e^x$

19. Solve the equation $\sin(e^x) = 0$.

$\sin(e^x) = 0$

$\implies e^x = n\pi$

$\implies X = \ln(n\pi), \quad n = 1, 2, \dots$

20. Show that $f(x) = e^{\cos(x)}$ is a periodic function. Find its domain, range and sketch the graph.

Recall that $-1 \leq \cos(x) \leq 1$

$\implies e^{-1} \leq e^{\cos(x)} \leq e^1$

$\implies$ dom f: $X \in \mathbb{R}$

ran f: $y \in [e^{-1}, e]$

Amplitude of f: $\dfrac{e - e^{-1}}{2}$

2π-periodic because of the periodicity of $\cos(X)$

$\implies$