MATH 125 Final Exam

May 13, 2015

Name: _____ **ID:** _____ **Rec:** ____

There are 18 problems in this exam, printed on 8 pages (not including this cover sheet). Make sure that you have them all.

The exam is in two parts: Part 1 consists of questions which should be *easy*. Getting at least 8 out of 12 correct on part 1 ensures a C or better on this exam.

If you have already passed part 1, you should skip it.
If you have NOT yet passed part 1, you MUST get 8/12 in order for part 2 to be graded.

Part I:

Question:	1	2	3	4	5	6	7	8	9	10	11	12	Total
Points:	1	1	1	1	1	1	1	1	1	1	1	1	12
Score:													

Part II:

Question:	13	14	15	16	17	18	Total
Points:	12	12	13	12	12	13	74
Score:							

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, calculators, extra papers, and discussions with friends are not permitted.**

Points will be taken off for writing mathematically false statements, even if the rest of the problem is correct.

Leave all answers in exact form (that is, do *not* approximate π, square roots, and so on.) Algebraic simplification is typically not necessary, unless it is.

You have 2 hours and 30 minutes to complete this exam. If you finish before that, it is advisable to use the remaining time to look back over your answers and make sure you still agree with them. If you *still* have more time, review all the choices you have made in your life up to this point, and make sure you still agree with them.

Part 1: Skip this part if you have passed part 1.
Otherwise you must pass it for part 2 to be graded. **Name:** _____

1 point 1. Find $f'(x)$ if $f(x) = 5\left(x^3 - \cos x + \sqrt{x}\right)^{10}$

1. _____

1 point 2. Evaluate $\lim_{x\to 0} \dfrac{\cos(3x)}{3x+6}$

2. _____

1 point 3. Evaluate $\lim_{h\to 0} \dfrac{3(x+h)^3 - 3x^3}{h}$.

3. _____

1 point 4. Find $\dfrac{dy}{dx}$ if $y = \dfrac{5x^3 - e^4}{\pi + \ln x}$

4. _____

1 point 5. Write the equation of the line tangent to $y = \ln x + \sin\left(\dfrac{\pi x}{2}\right)$ at $x = 1$.

5. _____

1 point 6. Compute the derivative with respect to t: $5t^5 - \dfrac{2t^2}{3} + \dfrac{5}{t} + \sin\left(\dfrac{\pi}{5}\right)$

6. _____

Part 1: Skip this part if you have passed part 1.
Otherwise you must pass it for part 2 to be graded. **Name:** _____

1 point 7. Find the x-coordinate of the point of inflection of $g(x) = 4x^3 + 18x^2 - 9x + 5$.

7. _____

1 point 8. What is the largest interval on which $f(x)$ is increasing if $f(x) = -2x^3 - 12x^2 + 72x + 100$?

8. _____

1 point 9. Find $P'(4)$ if $P(x) = \left(3\sqrt{x} - \cot\left(\frac{\pi x}{8}\right)\right)\left(\frac{x^2}{8} - 1\right)$.

9. _____

1 point 10. Compute $F'(t)$ if $F(t) = e^{\frac{10}{t}} + e^{10t}$.

10. _____

1 point 11. Find the x-coordinate of the local minimum of $8x^3 - 6x^2 - 12x + 5$. If there is none, write "None".

11. _____

1 point 12. Compute the derivative of $\frac{\cos(10x)}{7} - \arcsin(10x)$.

12. _____

Part 2: These will be graded **only** if you have passed part 1. **Name:** _____

13. Find an antiderivative (that is, a function whose derivative is the given function) for each of the following functions:

3 points (a) $f(x) = 2x^3 - 6x^2 + 8x + e^2$

(a) _____

3 points (b) $g(x) = e^{2x} + \sin(3x)$

(b) _____

3 points (c) $h(x) = 5\sqrt{x} - \dfrac{3}{x}$

(c) _____

3 points (d) $k(x) = \dfrac{5}{1+x^2}$

(d) _____

Part 2: These will be graded **only** if you have passed part 1. **Name:** _____

12 points

14. A company plans to build a pipeline from its drilling station, which is located in the ocean 2 miles south from a straight shoreline running east-west, to a pumping station which is located 6 miles east from the point on the shore directly opposite the drilling station. The pipeline will cost \$600/mile to run under the water and \$200/mile to run under the land. Where should the pipeline intersect the shore to be built for the minimum cost?

Part 2: These will be graded **only** if you have passed part 1. **Name:** _____

8 points 15. (a) For the curve given by $x^3 - 3y^4 = 4x^2y^3 - 6$, find dy/dx when $x = 1$ and $y = 1$.

5 points (b) Use your answer to the previous part to estimate the y-value of a point on the curve with $x = 1.2$.

Part 2: These will be graded **only** if you have passed part 1. **Name:** _____

16. Consider the function $f(x) = 4x^5 + 5x^4 - 40x^3$.

4 points (a) Find the x-values of all critical points of $f(x)$

4 points (b) State the largest interval on which $f(x)$ is decreasing.

4 points (c) Give the x-values at which the absolute maximimum and absolute minimum values of f occur when $-1 \leq x \leq 3$.

Part 2: These will be graded **only** if you have passed part 1. **Name:** _____

12 points

17. Sand is falling from a chute at the rate of 144ft^3 per minute, and is forming a conical pile whose diameter is always three times its height. Find the rate at which the height of the pile is growing when the pile is 1 foot high. You might find it useful to know that the volume of a cone of radius r and height h is $\pi r^2 h/3$, its surface area is $\pi r\left(r+\sqrt{r^2+h^2}\right)$, or that ice cream cone was invented in 1896 by Italo Marchiony. Or maybe not.

18. Let $R(x) = \dfrac{e^{2x} - 1}{\pi x}$.

8 points

(a) Find a value k so that if we define $R(0) = k$, the resulting function is continuous. Fully justify your answer.

5 points

(b) Is the function in the previous part differentiable at all values of x? Fully justify your answer.