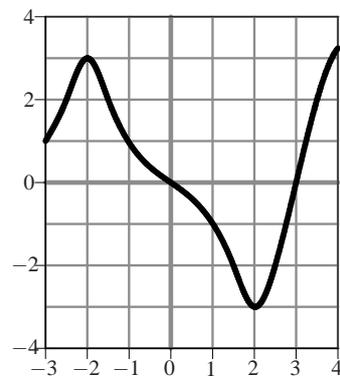5. At right is the graph of **the derivative** f' of a function.

(a) 4 points List all values of x with $-3 \le x \le 4$ where $f(x)$ has a local maximum.

> **Solution:** A local maximum for $f(x)$ will occur where $f'(x)$ changes from positive to negative. This happens at $x = 0$.

(b) 4 points At $x = -1$, is $f(x)$ concave up, concave down, or neither?

> **Solution:** We know that a function is concave up when its second derivative is positive, and concave down when f'' is negative. The graph shows $f'(x)$, which is decreasing near $x = -1$. That means the derivative of $f'(x)$ is negative near $x = -1$, so $f''(-1) < 0$. Hence $f(x)$ is concave down at $x = -1$.

6. 16 points For each of the 4 functions graphed in the left column, find the corresponding derivative function among any of the 8 choices on the right (not just on the same row) and put its letter in the corresponding box.
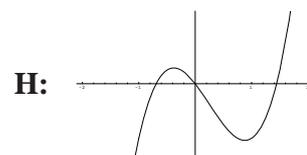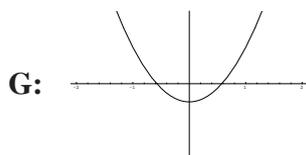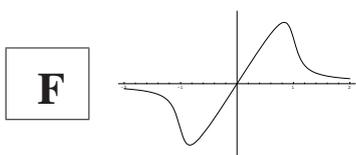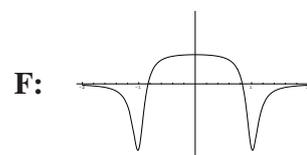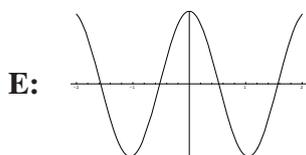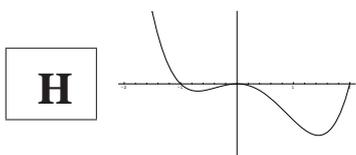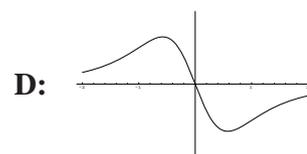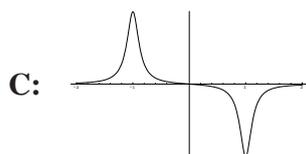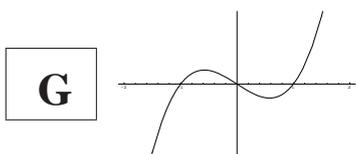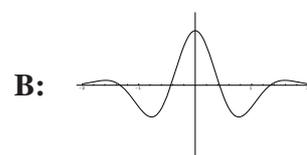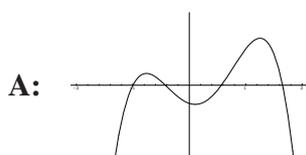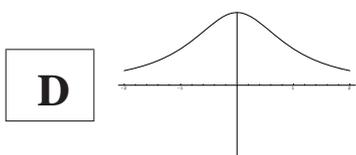
6. At right is the graph of **the derivative** $f'(x)$ of a function $f(x)$. Use it to answer each of the following questions.
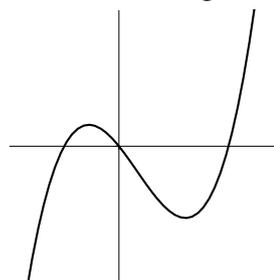
4 points

(a) Is $f(x)$ concave up, concave down, or neither at $x = 0$?

Solution: Since the derivative is decreasing at $x = 0$, we know $f(x)$ is concave down there.

4 points

(b) Which of the following best represents the graph of $f(x)$? (circle your answer).
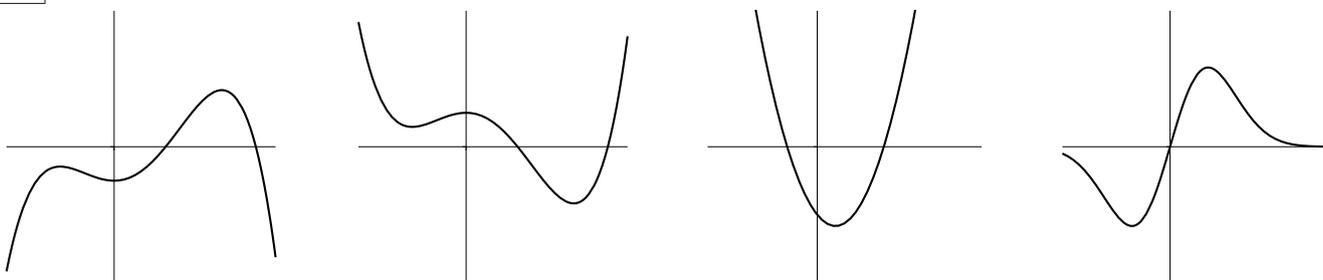
Solution: The graph of $f(x)$ is
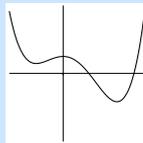
4 points

(c) Which of the following best represents the graph of $f''(x)$? (circle your answer).
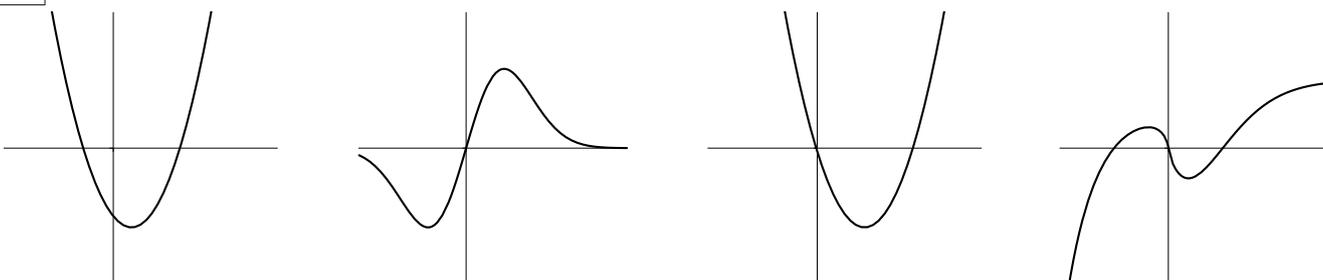
Solution: The graph of $f''(x)$ is
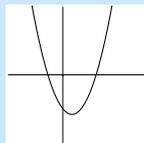
8 points 3. In the paragraph below is a description of how the amount of water $W(t)$ in a tub varied with time.

> The tub held about 50 gallons of green, brackish water, with some stuff floating in it that I didn't even want to guess about. I had to get it out of there. When I opened the drain the water drained out rapidly at first, but then it went slower and slower, until it stopped completely after about 5 minutes. The tub was about $1/4$-full of that nasty stuff. Would I have to stick my hand in it? *Ick*— there was no way I could do that. I just stared at it for a couple of minutes, but then I got an idea. I dumped in about 10 gallons of boiling water. That did something: there was this tremendous noise like *BLUUUUURP*, and then the tub drained steadily, emptying completely in just a minute or so.

Use this description to sketch a graph of $W(t)$ and its derivative $W'(t)$. Pay careful attention to slope and concavity. Label the axes, with units.

Solution: A pair of graphs something like those below agrees with the description (the graph of $W(t)$ is on the left, its derivative on the right). The graph starts out at 50, then decreases "slower and slower", (which is another way of saying it is decreasing and concave up) until it finally flattens out at about 5 minutes with a value of $12\frac{1}{2}$. The "spike" at around 7 minutes corresponds to when the 10 gallons of boiling water were added, raising the amount to $22\frac{1}{2}$, and then the level drops with constant slope, hitting the axis just about a minute later.
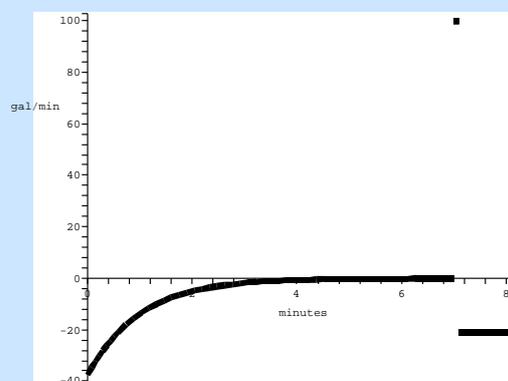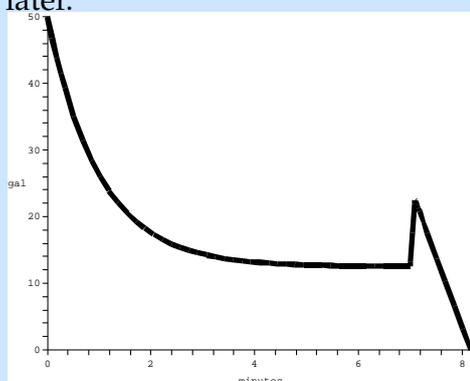
Of course, you might have minor variations. For example, the region around 7 minutes could be smooth, or discontinuous.

For the derivative, it should be negative everywhere except right around 7 minutes when the boiling water is added. The part before 7 minutes should be concave down and increasing, limiting on zero. In the version above, the short steep segment corresponds to adding the 10 gallons very quickly (at 100 gal/min), then the immediate quick draining is a jump back down to a fast rate of -20 gal/minute. If you made your graph of $W(t)$ smooth, the derivative should be continuous, with a quick bump going way up and then rapidly back down to -20 or so.

7. At right is the graph of **the derivative** $g'(x)$ of a function $g(x)$. Use it to answer each of the following questions.

List all values of x in the interval $[-5, 5]$ where $g(x)$ has a local maximum.

2 points

(a) **Solution:** A local maximum for $g(x)$ will occur where $g'(x) = 0$ and $g'(x)$ changes from positive to negative. This happens when $x = 0$.
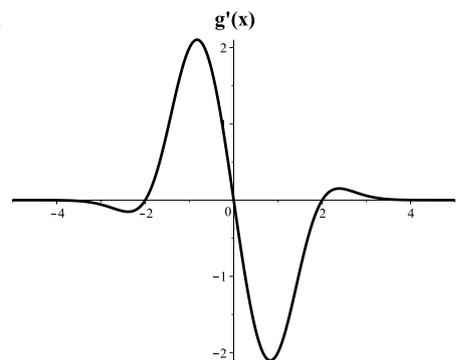
2 points

(b) List all values of x in the interval $[-5, 5]$ where $g(x)$ has a local minimum.

Solution: We get a local minimum when $g'(x) = 0$ and $g'(x)$ changes from negative to positive. Thus, $x = -2$ or $x = 2$.

4 points

(c) Assuming that the $g'(x)$ behaves the same for $x > 5$ as it does for $4 < x < 5$, which of the following should be true (circle your answer)?

    A. $\lim\limits_{x \to \infty} g(x) = +\infty$

    B. $\lim\limits_{x \to \infty} g(x)$ is a finite number

    C. $\lim\limits_{x \to \infty} g(x) = -\infty$

    D. $\lim\limits_{x \to \infty} g(x)$ does not exist

    E. $\lim\limits_{x \to \infty} g(x)$ can not be determined from this information

WHY? Justify your answer below. No credit without a justification.

Solution: The correct answer is " $\lim\limits_{x \to \infty} g(x)$ is a finite number." In this graph, for $x > 4$, $g'(x)$ is zero. This means that the tangent line to $g(x)$ is horizontal for large x. Thus, the limit is a constant.

If you thought that the value of $g'(x)$ was just tending to zero, the correct answer is either B (if it tends to zero very fast), A (if it tends to zero slowly, but remains positive as it does so, or "cannot be determined".

Which answer gets full credit depends on your explanation, and whether your explanation matches your choice.