Solutions

(1) $f'(x) = \dfrac{(2x^3+1)(2x-5) - (x^2-5x+3)(6x^2)}{(2x^3+1)^2}$

(2) $f'(x) = \dfrac{(5+2\sec x)(\sin x) - (4-\cos x)(2\sec x \tan x)}{(5+2\sec x)^2}$

(3) $f'(x) = \dfrac{(x^2 - \sqrt[3]{x})(8 + \frac{1}{3}x^{-\frac{2}{3}}) - (8x + \sqrt[3]{x})(2x - \frac{1}{3}x^{-\frac{2}{3}})}{(x^2 - \sqrt[3]{x})^2}$

(4) $f'(x) = (8x^4 - 7x^3 + 2x)(8x - 5) + (4x^2 - 5x)(32x^3 - 21(x^2+2)$

(5) $f'(x) = (\sqrt{x} + 3x^2 - 5)(27x^2 - 11) + (9x^3 - 11x)(\frac{1}{2\sqrt{x}} + 6x)$

(6) $f'(x) = 4\tan^3\left(\frac{3x-1}{x^2}\right)\sec^2\left(\frac{3x-1}{x^2}\right)\left(\dfrac{(x^2)(3) - (3x-1)(2x)}{x^4}\right)$

(7) $f'(x) = \dfrac{1}{1-(e^x)^2} \cdot e^x$

(8) $f'(x) = e^{3\sec x} \cdot 3\sec x \tan x$

(9) $f'(x) = \dfrac{1}{1+\sin^2 x} \cdot \cos x$

(10) $f'(x) = \dfrac{12x^2+8}{4x^3+8x}$

(11) $f'(x) = \dfrac{-\csc x \cot x}{1+\csc x}$

(12) $f'(x) = \dfrac{1}{2\sqrt{\frac{4x-2}{5x+1}}} \cdot \dfrac{(5x+1)(4)-(4x-2)(5)}{(5x+1)^2}$

(13) $\dfrac{dy}{dx} = \sec^2(3x) \cdot 3$

at $x = \frac{\pi}{12}$, $\dfrac{dy}{dx} = \sec^2\left(\frac{\pi}{4}\right) \cdot 3 = 6$

at $x = \frac{\pi}{12}$, $y = \tan\left(\frac{\pi}{4}\right) = 1$

$y - 1 = 6\left(x - \frac{\pi}{12}\right)$

(14) $\frac{dy}{dx} = 2xe^{x^2}$

at $x=1$, $\frac{dy}{dx} = 2e$ at $x=1$ $y=e$

$y-e = 2e(x-1)$

(15) $\frac{dy}{dx} = -\pi \sin(\pi x)$

at $x=\frac{1}{6}$, $\frac{dy}{dx} = -\pi \sin\left(\frac{\pi}{6}\right) = -\frac{\pi}{2}$

at $x=\frac{1}{6}$, $y = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$y-\frac{\sqrt{3}}{2} = -\frac{\pi}{2}\left(x-\frac{1}{6}\right)$

(16) $3x^2 + 3y^2\frac{dy}{dx} + 1 = 0$

$3(1^2) + 3(1^2)\frac{dy}{dx} + 1 = 0$

$\frac{dy}{dx} = -\frac{4}{3}$

$y-1 = -\frac{4}{3}(x-1)$

(7) $2x\frac{dy}{dx} + 2y - 4y^3\frac{dy}{dx} = 0$

$2(1)\frac{dy}{dx} + 2(1) - 4(1^3)\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 1 \qquad y - 1 = 1(x-1)$

(8) $\frac{dy}{dx} = \frac{5}{5x-4}$

at $x=1 \quad \frac{dy}{dx} = 5 \qquad y - 0 = 5(x-1)$

(9) $\frac{dy}{dx} = \frac{1}{3}(3x+5)^{-\frac{2}{3}} \cdot 3 = \frac{1}{(\sqrt[3]{3x+5})^2}$

at $x=1, \frac{dy}{dx} = \frac{1}{4}$

at $x=1, \; y = \sqrt[3]{8} = 2$

$y - 2 = \frac{1}{4}(x-1)$

(10) $\frac{dy}{dx} = \frac{2}{x^2}$

at $x=1, \frac{dy}{dx} = 2$

at $x=1, \; y = 4-2 = 2$

$y - 2 = 2(x-1)$

(26) $4x^3 + 3x^2 \cdot 2y\frac{dy}{dx} + 6xy^2 - 2y\frac{dy}{dx} = 0$

$4x^3 + 6xy^2 = 2y\frac{dy}{dx} + 6x^2y\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{4x^3 + 6xy^2}{2y + 6x^2y}$

(27) $6x^2 + 7x \cdot \frac{dy}{dx} + 7y + 6y\frac{dy}{dx} = 0$

$6x^2 + 7y = -7x\frac{dy}{dx} - 6y\frac{dy}{dx}$

$\frac{6x^2 + 7y}{-7x - 6y} = \frac{dy}{dx}$

(28) $2x + 2y\frac{dy}{dx} = 4x\frac{dy}{dx} + 4y$

$2y\frac{dy}{dx} - 4x\frac{dy}{dx} = 4y - 2x$

$\frac{dy}{dx} = \frac{4y - 2x}{2y - 4x}$

(29) $e^{x+y}\left(1 + \frac{dy}{dx}\right) = 2x$

$1 + \frac{dy}{dx} = \frac{2x}{e^{x+y}}$

$\frac{dy}{dx} = \frac{2x}{e^{x+y}} - 1$

(30) $\cos(xy)\left(x\frac{dy}{dx}+y\right)=12x^2$

$x\frac{dy}{dx}+y=\frac{12x^2}{\cos xy}$

$x\frac{dy}{dx}=\frac{12x^2}{\cos xy}-y$

$\frac{dy}{dx}=\frac{12x^2}{x\cos xy}-\frac{y}{x}$

(31) $\frac{dy}{dx}=2x-8$ x-axis has slope=0

set $2x-8=0$

$x=4$

(32) $\frac{dy}{dx}=8x+16$ $y=4x-2$ has slope=4

set $8x+16=4$

$8x=-12$

$x=\frac{-12}{8}=-\frac{3}{2}$

(33) $\frac{dy}{dx}=3x^2-24x+30$ $y=9x+3$ has slope=9

set $3x^2-24x+30=9$

$3x^2-24x+21=0$

$x^2-8x+7=0$

$(x-7)(x-1)=0$

$x=1,7$

(39) $\frac{dy}{dx} = 6x^2 + 18x - 20$ $y - 4x = 1$ has slope $= 4$

set $6x^2 + 18x - 20 = 4$

$6x^2 + 18x - 24 = 0$

$x^2 + 3x - 4 = 0$

$(x+4)(x-1) = 0$

$x = -4, 1$

(35) $\frac{dy}{dx} = 10x + 8$ x-axis has slop = 0

set $10x + 8 = 0$

$x = \frac{-8}{10} = \frac{-4}{5}$