MAT125 Spring 2014 Midterm #2

Your name:____________________________

TA's name:____________________________

Problem #1: Find the derivative of each function.

a) $f(x) = \frac{3-\sqrt{x}}{3+\sqrt{x}}$

$$f'(x) = \frac{(3+\sqrt{x})\left(-\frac{1}{2\sqrt{x}}\right) - (3-\sqrt{x})\left(\frac{1}{2\sqrt{x}}\right)}{(3+\sqrt{x})^2}$$

b) $f(x) = (3x^2 + 9x - 4)(4x^3 + x^2 - x)$

$$f'(x) = (3x^2+9x-4)(12x^2+2x-1) + (4x^3+x^2-x)(6x+9)$$

Problem #2: Find the equation of the tangent line to $y = \sin(4x)$ at $x = \frac{\pi}{16}$.

$$y = \sin\left(4\left(\frac{\pi}{16}\right)\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$y - \frac{\sqrt{2}}{2} = m\left(x - \frac{\pi}{16}\right)$$

$$\frac{dy}{dx} = \cos(4x)(4)$$

$$\text{at } x = \frac{\pi}{16} : \frac{dy}{dx} = \cos\left(4\left(\frac{\pi}{16}\right)\right)(4) = 4\left(\frac{\sqrt{2}}{2}\right) = 2\sqrt{2}$$

$$y - \frac{\sqrt{2}}{2} = 2\sqrt{2}\left(x - \frac{\pi}{16}\right)$$

Problem #3. Find all x-values of

$$f(x) = x^3 - 6x^2 - 36x + 9$$

for which either $f'(x) = 0$ or $f'(x)$ is not defined.

$f'(x) = 3x^2 - 12x - 36 = 0$

$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

$x = 6 \quad x = -2$

Problem #4: Find $\frac{dy}{dx}$ if $x^3 - 5xy^2 + y^3 = 1$.

$$3x^2 - \left[5x\left(2y\frac{dy}{dx}\right) + 5y^2\right] + 3y^2\frac{dy}{dx} = 0$$

$$3x^2 - 10xy\frac{dy}{dx} - 5y^2 + 3y^2\frac{dy}{dx} = 0$$

$$3x^2 - 5y^2 = 10xy\frac{dy}{dx} - 3y^2\frac{dy}{dx}$$

$$3x - 5y^2 = \frac{dy}{dx}\left(10xy - 3y^2\right)$$

$$\frac{3x - 5y^2}{10xy - 3y^2} = \frac{dy}{dx}$$

Problem #5: Find the equation of the tangent line to $\ln(2x^2 - y^2) = 0$ at (1,1).

$$y-1 = m(x-1)$$

$$\frac{4x - 2y\frac{dy}{dx}}{2x^2 - y^2} = 0$$

$$\frac{4 - 2\frac{dy}{dx}}{2-1} = 0$$

$$4 - 2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = 2$$

$$y-1 = 2(x-1)$$

Problem #6: Find $\frac{dy}{dx}$ if:

(a) $y = \tan^{-1}(2x)$

$$\frac{dy}{dx} = \frac{1}{1+(2x)^2} \cdot 2 = \frac{2}{1+4x^2}$$

b) $f(x) = \sin^3\left(\frac{2-5x}{x^2}\right)$

$$f'(x) = 3\sin^2\left(\frac{2-5x}{x^2}\right)\left[\frac{x^2(-5) - (2-5x)(2x)}{x^4}\right]$$

Problem #7: Find the points (x, y) where the line tangent to

$y = x^3 - 6x^2 - 30x + 4$ is parallel to $15x + y = 10$.

$$15x + y = 10$$
$$y = -15x + 10$$
$$\text{slope} = -15$$

$$\frac{dy}{dx} = 3x^2 - 12x - 30$$

parallel so slopes are equal

$$3x^2 - 12x - 30 = -15$$
$$3x^2 - 12x - 15 = 0$$
$$x^2 - 4x - 5 = 0$$
$$(x-5)(x+1) = 0$$
$$x = 5, \quad x = -1$$

$$y = 5^3 - 6(5^2) - 30(5) + 4$$
$$= 125 - 150 - 150 + 4$$
$$= -171$$
$$(5, -171)$$

$$y = (-1)^3 - 6(-1)^2 - 30(-1) + 4$$
$$= -1 - 6 + 30 + 4$$
$$= 27$$
$$(-1, 27)$$

Problem #8. Find all values of x where $y = x^2e^x$ has an absolute maximum or minimum on the interval $[-3, 1]$.

Take the derivative, set it to zero, solve to get critical points at x=0 and x=-2:

$$f'(x) = 2xe^x + x^2e^x = xe^x(x+2)$$

Then, check f(-3), f(-2), f(0), and f(1).

f(-3) = 9*e^-3
f(-2) = 4*e^-2 $= 4e/e^3 > 9/e^3$
f(0) = 0
f(1) = e

Observe that f(0) < f(-3) < f(-2) < f(1), so the absolute min is at x=0 and the max at x=1.

(there is a local maximim at x=-2, but the question doesn't ask for that).

It doesn't ask for the graph, either, but it looks like this: