MATH 125 First Midterm

October 13, 2009

Name: _____   **ID:** _____   **Rec:** _____

Question:	1	2	3	4	5	6	7	8	Total
Points:	9	9	12	8	8	12	8	8	74
Score:									

There are 8 problems in this exam. Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, calculators, extra papers, and discussions with friends are not permitted.** Telepathic communication with mathematically talented ducks is allowed, although you must also turn in the duck for grading along with your exam (geese and other fowl are not allowed).

Use non-erasable pen (not red) if you want to be able to contest the grading of any problems. Questions with erasures will not be regraded.

Leave all answers in exact form (that is, do *not* approximate π, square roots, and so on.)

You have 90 minutes to complete this exam.

1. Compute each of the following limits. If the limit is not a finite number, please distinguish between $+\infty$, $-\infty$, and a limit which does not exist (DNE). Justify your answer, at least a little bit.

3 points (a) $\displaystyle\lim_{x\to 2} \frac{x^2-4}{5x(x-2)}$

3 points (b) $\displaystyle\lim_{x\to\infty} 2\cos\left(\frac{\pi}{x}\right)$

3 points (c) $\displaystyle\lim_{x\to 4} \frac{x^2}{(x-4)^2}$

2. More of the same: compute each of the following limits. If the limit is not a finite number, please distinguish between $+\infty$, $-\infty$, and a limit which does not exist (DNE). Justify your answer, at least a little bit.

3 points   (a) $\lim\limits_{x\to\infty} \dfrac{x^2-9}{5x(x-3)}$

3 points   (b) $\lim\limits_{h\to 2} \dfrac{(x+h)^2-x^2}{h}$

3 points   (c) $\lim\limits_{x\to-\infty} e^x\cos(x)$

3. Let $f(x) = 5x^3 - 8x + 2$.

| 3 points |

(a) Find the slope of the secant line passing through the points on the curve $y = f(x)$ where $x = 0$ and $x = 1$.

| 3 points |

(b) Find $f'(1)$.

| 3 points |

(c) Write the equation of the tangent line to the graph of $y = f(x)$ when $x = 1$.

| 3 points |

(d) At $x = 1$, is $f(x)$ concave up, concave down, or neither? Justify your answer fully.
This problem requires material we haven't covered yet.

8 points   4. For what values of x is the function $f(x) = \dfrac{e^x}{2 - e^{1/x}}$ continuous?
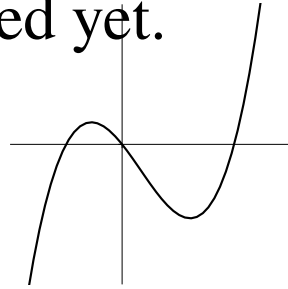
8 points   5. Write a limit that represents the slope of the graph

$$y = \begin{cases} |x|^x & x \neq 0 \\ 1 & x = 0 \end{cases}$$

at $x = 0$. You **do not need to evaluate the limit.**

This problem uses material not covered yet.

6. At right is the graph of **the derivative** $f'(x)$ of a function $f(x)$. Use it to answer each of the following questions.
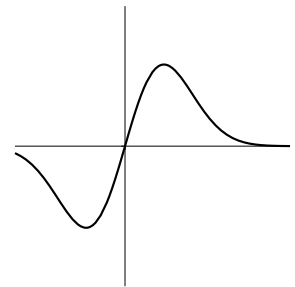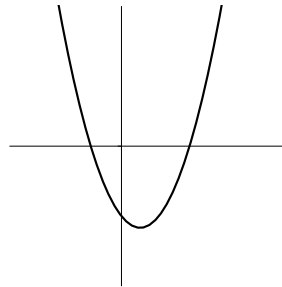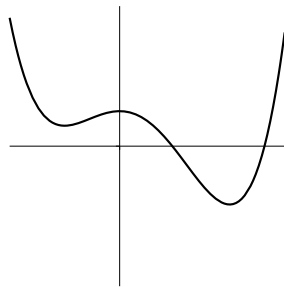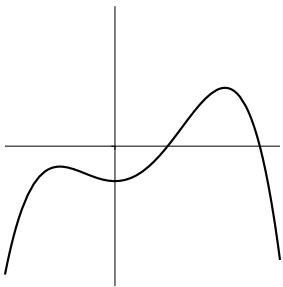
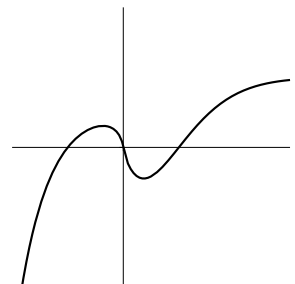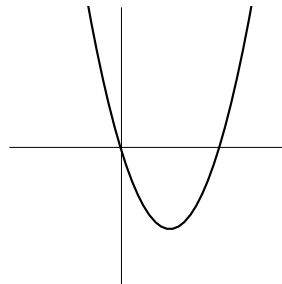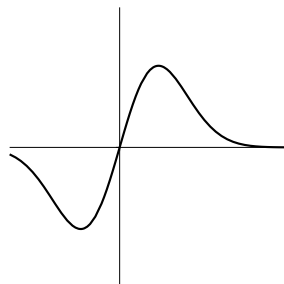4 points   (a) Is $f(x)$ concave up, concave down, or neither at $x = 0$?

4 points   (b) Which of the following best represents the graph of $f(x)$? (circle your answer).

4 points   (c) Which of the following best represents the graph of $f''(x)$? (circle your answer).
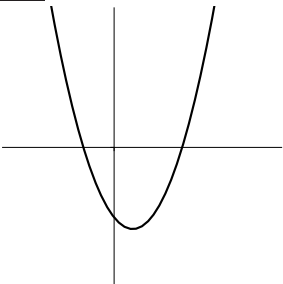
7. Let $f(x) = \dfrac{x^2 - 2x}{5(x^2 - 4)}$

| 4 points |

    (a) Identify the horizontal asymptotes of $f(x)$. If there are none, write "NONE".

| 4 points |

    (b) Identify the vertical asymptotes of $f(x)$. If there are none, write "NONE".

| 8 points | 8. Write a function which expresses the area of a rectangle with a perimeter of 10 feet in terms of its width.

MATH 125   First Midterm

February 26, 2015

Name: _____   **ID:** _____   **Rec:** _____

Question:	1	2	3	4	5	6	7	8	Total
Points:	9	9	8	8	8	9	8	10	69
Score:									

There are 8 problems in this exam. Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, calculators, extra papers, and discussions with friends are not permitted.** If you engraved any notes on a piece of ice, you may use them during the exam. However, be sure not to get your exam, desk, or neighbor wet when your notes begin to melt. This offer will not be valid during the second midterm, since ice should be less prevalent in late March.

Points will be taken off for writing mathematically false statements, even if the rest of the problem is correct.

Use non-erasable pen (not red) if you want to be able to contest the grading of any problems. Questions with erasures will not be regraded.

Leave all answers in exact form (that is, do *not* approximate π, square roots, and so on.)

You have 90 minutes to complete this exam.

1. Compute each of the following limits. If the limit is not a finite number, please distinguish between $+\infty$, $-\infty$, and a limit which does not exist (DNE). Justify your answer, at least a little bit.

3 points   (a) $\displaystyle\lim_{x \to 4} \frac{x^2 - 16}{5x(x - 4)}$

3 points   (b) $\displaystyle\lim_{x \to 3} e^x \ln(x)$

3 points   (c) $\displaystyle\lim_{x \searrow 4^+} \frac{x^2 - 9}{(x - 4)}$

2. More of the same: compute each of the following limits. If the limit is not a finite number, please distinguish between $+\infty$, $-\infty$, and a limit which does not exist (DNE). Justify your answer, at least a little bit.

3 points   (a) $\lim\limits_{x \to +\infty} \sqrt{9x^2 + x} - 3x$

3 points   (b) $\lim\limits_{h \to -1} \dfrac{(x + h)^2 - x^2}{h}$

3 points   (c) $\lim\limits_{x \to 0} \dfrac{\tan(\pi x)}{\sin(\pi x)}$

8 points 3. In the paragraph below is a description of how the amount of water $W(t)$ in a tub varied with time.

> The tub held about 50 gallons of green, brackish water, with some stuff floating in it that I didn't even want to guess about. I had to get it out of there. When I opened the drain the water drained out rapidly at first, but then it went slower and slower, until it stopped completely after about 5 minutes. The tub was about $1/4$-full of that nasty stuff. Would I have to stick my hand in it? *Ick—* there was no way I could do that. I just stared at it for a couple of minutes, but then I got an idea. I dumped in about 10 gallons of boiling water. That did something: there was this tremendous noise like *BLUUUUURP,* and then the tub drained steadily, emptying completely in just a minute or so.

Use this description to sketch a graph of $W(t)$ and its derivative $W'(t)$. Pay careful attention to slope and concavity. Label the axes, with units.

8 points 4. What value of k is necessary so that the function

$$f(x) = \begin{cases} kx + 5 & x < 3 \\ x^2 - x & x \geq 3 \end{cases}$$

is continuous for all values of x? Justify your answer fully.

8 points 5. Write a limit that represents the slope of the graph

$$y = \begin{cases} |x|^{\sin x} & x \neq 0 \\ 1 & x = 0 \end{cases}$$

at $x = 0$. You **do not need to evaluate the limit.**

6. Let $f(x) = 6x^2 - 9x + 5$.

3 points
    (a) Find the slope of the secant line passing through the points on the curve $y = f(x)$ where $x = 0$ and $x = 1$.
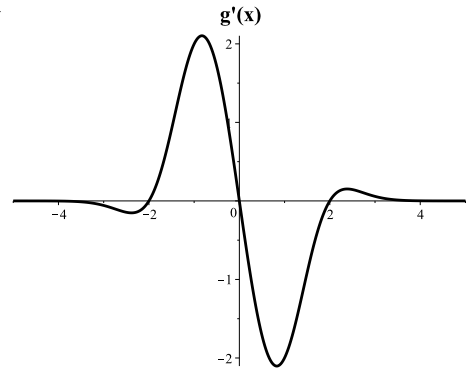
3 points
    (b) Find $f'(1)$.

3 points
    (c) Write the equation of the tangent line to the graph of $y = f(x)$ when $x = 1$.

Material in next problem not covered yet.

7. At right is the graph of **the derivative** $g'(x)$ of a function $g(x)$. Use it to answer each of the following questions.

g'(x)

$\boxed{\text{2 points}}$ (a) List all values of x in the interval $[-5, 5]$ where $g(x)$ has a local maximum.

$\boxed{\text{2 points}}$ (b) List all values of x in the interval $[-5, 5]$ where $g(x)$ has a local minimum.

$\boxed{\text{4 points}}$ (c) Assuming that the $g'(x)$ behaves the same for $x > 5$ as it does for $4 < x < 5$, which of the following should be true (circle your answer)?

      A. $\lim\limits_{x \to \infty} g(x) = +\infty$

      B. $\lim\limits_{x \to \infty} g(x)$ is a finite number

      C. $\lim\limits_{x \to \infty} g(x) = -\infty$

      D. $\lim\limits_{x \to \infty} g(x)$ does not exist

      E. $\lim\limits_{x \to \infty} g(x)$ can not be determined from this information

WHY? Justify your answer below. No credit without a justification.

10 points   8. Sketch the graph of a function $f(x)$ which satisfies all of the following properties:

- $f(2) = 1$
- $\lim_{x \to 1} f(x) \neq f(1)$
- $\lim_{x \to 2} f(x) = 0$
- $\lim_{x \searrow 0^+} f(x) = -\infty$

- $\lim_{x \nearrow 0^-} f(x) = +\infty$
- $\lim_{x \to +\infty} f(x) = 0$
- $\lim_{x \to -\infty} f(x) = +\infty$
- $f'(-1) = 0$

Be sure to include axes and label important values.