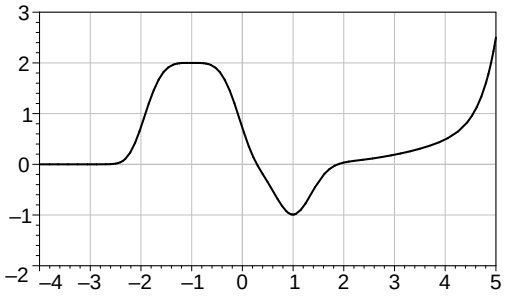
Review for Second Midterm Exam MAT 125

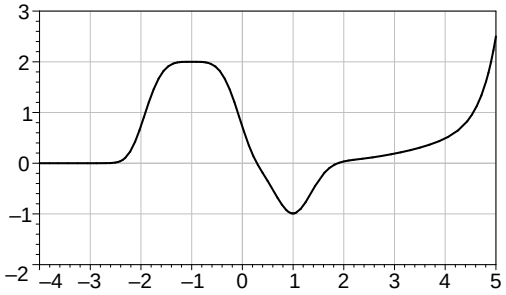
Here are some problems you can use to prepare yourself for the second exam. Note that this is not an exhaustive set of problems: just because something is here doesn't mean it will be on the exam, and there may be material on the exam not represented here. You should not need a calculator to do any of these problems.

The exam will be held on Tuesday, March 20, at 8:30 PM. Do not forget to bring your student ID card or another photo ID like a driver's license.

1. At right is a graph of a function $f(x)$.
Draw a graph of the derivative $f'(x)$.
At which x values is f increasing?
At which x values is f concave up?



2. At right is a graph of the **derivative** $g'(x)$ of a function.
Draw the graph of a function g which has the given graph as its derivative.
At which x values is g increasing?
At which x values is g concave up?



3. For each of the following functions $f(x)$, compute the derivative $f'(x)$.

a. $f(x) = x^5\left(x^3 - \dfrac{1}{x^2}\right)$

b. $f(x) = e^{2x} - 2x^e$

c. $f(x) = \left(\dfrac{x^2}{\tan(2x)}\right)^3$

d. $f(x) = \cos\left(\sqrt{x}\right)$

e. $f(x) = e^{(x^2-\sqrt{x})}$

f. $f(x) = \sqrt{\dfrac{5}{x} - \dfrac{x}{5}}$

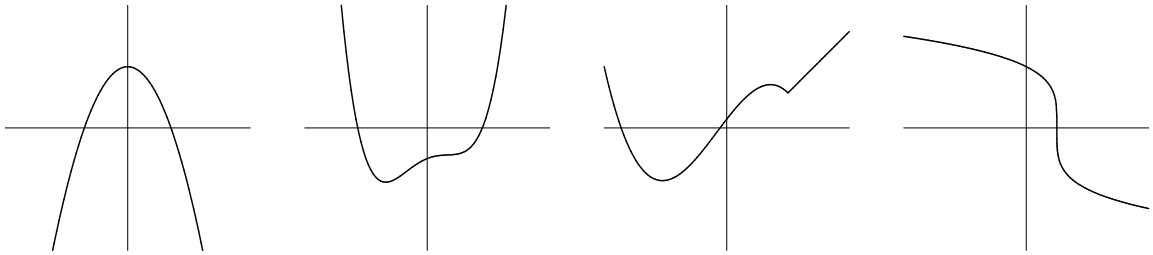
g. $f(x) = \cos(x)\sin(x)$

4. Let $f(x) = x\cos(\pi x) + 1$.

a. Compute $f'(x)$ and find the formula of the tangent line to the graph of $f(x)$ through the point $(1, 0)$.

b. Compute $f''(2)$. Is $f(x)$ concave up or concave down at $x = 2$? Justify your answer.

5. The graphs of several functions $f(x)$ are shown below. On the same set of axes, sketch the function $f'(x)$.



6. Let $f(x) = 2x^2 + xe^x$.

a. Compute $f'(x)$ and $f''(x)$.

b. For which of the following values of x, is $f(x)$ increasing near x? -1, 0, 2.

c. Is $f(x)$ concave up near $x = 0$?

d. Are there any values of x for which $f(x)$ is concave down near x?
(**Hint:** Remember that $e^t > 0$ for all values of t).

7. Which of the following represents $f'(2)$ where $f(x) = e^{x^2}$.

$$\lim_{x\to 2} \frac{e^{x^2} - e^{a^2}}{h} \qquad \lim_{h\to 0} \frac{e^4(e^{4h+h^2} - 1)}{h} \qquad \lim_{x\to 2} \frac{e^{x^2} - e^2}{x - 2} \qquad \lim_{h\to 0} \frac{e^{(x^2+h^2)} - e^{x^2}}{h}$$

8. If $h(0) = 1$, $h'(0) = 3$, $h(1) = 0$, $h'(1) = -1$, $h(2) = 0$, $h'(2) = 2$, and $f(x) = h(h(x))$, what is $f'(1)$?

9. In the paragraph below is a description of how the amount of water $W(t)$ in a tub varied with time.

> The tub held about 50 gallons of green, brackish water, with some stuff floating in it that I didn't even want to guess about. I had to get it out of there. When I opened the drain the water drained out rapidly at first, but then it went slower and slower, until it stopped completely after about 5 minutes. The tub was about 1/4-full of that nasty stuff. Would I have to stick my hand in it? *Ick*— there was no way I could do that. I just stared at it for a couple of minutes, but then I got an idea. I dumped in about 10 gallons of boiling water. That did something: there was this tremendous noise like *BLUUUUURP*, and then the tub drained steadily, emptying completely in just a minute or so.

Use this description to sketch a graph of $W(t)$ and its derivative $W'(t)$. Pay careful attention to slope and concavity. Label the axes, with units.