SAMPLE MIDTERM 1
MAT 125 Spring 2004
Midterm 1 is 8:30-10:00pm,
Monday, 2/23/04

1. Place the letter corresponding to the correct answer in the box next to each question. Each question is worth 1 point.

(i) [d] Suppose f is a linear function such that $f(1) = -1$ and $f(3) = 2$. Then $f(4) =$?
(a) 3 **(b)** $3\frac{1}{3}$ **(c)** $3\frac{2}{3}$ **(d)** $3\frac{1}{2}$ **(e)** $3\frac{3}{4}$ **(f)** none of these.

SINCE f IS LINEAR, $f(x) = \frac{3}{2}(x-1) - 1$, SO $f(4) = \frac{9}{2} - 1 = 3\frac{1}{2}$

(ii) [e] Simplify $\log_2(4x^2 2^x)$ **(a)** $\ln 2 + 2\ln x + x$ **(b)** $\log_2 2 + 2\log_2 x + x$ **(c)** $\ln 4 + 2\ln x + x\ln 2$ **(d)** $2 + 2\ln x + x$ **(e)** $2 + 2\log_2 x + x$ **(f)** none of these.

$\log_2(4) + 2\log_2 x + x\log_2 2 = 2 + 2\log_2 x + x \cdot 1$

x	$\ln(x)$
2	.69
3	1.10
4	1.39
5	1.61

(iii) [e] Using the infomation in the table, estimate $\log_3 5$.
(a) 1.16 **(b)** 1.26 **(c)** .68 **(d)** .86 **(e)** 1.46 **(f)** none of these.

$\log_3 5 = \frac{\ln 5}{\ln 3} = \frac{1.61}{1.10} \approx 1.46$

(iv) [a] Express the following function as an explicit formula: take a number and add 1 to it; then square the result and multiply by 4. **(a)** $f(x) = 4(x+1)^2$ **(b)** $f(x) = (4x+1)^2$ **(c)** $f(x) = 4x^2 + 1$ **(d)** $f(x) = (4x)^2 + 1$ **(e)** $f(x) = 4(x^2+1)$ **(f)** none of these.

(v) [c] The function $h(x) = x^2 + 1$ is a composition of the form $h(x) = f(g(x))$ where **(a)** $f(x) = x^2$ and $g(x) = 1$ **(b)** $f(x) = x^2$ and $g(x) = x + 1$ **(c)** $f(x) = x + 1$ and $g(x) = x^2$ **(d)** $f(x) = x$ and $g(x) = x^2$ **(e)** $f(x) = (x+1)^2$ and $g(x) = -2x$ **(f)** none of these.

FIRST SQUARE x $(g(x))$, THEN ADD 1 $(f(x))$

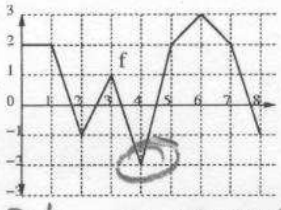
(vi) [d] Which of the following functions is not defined for all real numbers? **(a)** $y = \sin(x^2)$ **(b)** $\ln(1+x^2)$ **(c)** $y = e^{\cos(x)}$ **(d)** $y = \ln(\sin(x))$ **(e)** $y = \sqrt{\cos(x)+2}$ **(f)** none of these.

SINCE $\sin x < 0$ FOR SOME x, AND DOMAIN OF $\ln x$ IS $x > 0$.

(vii) [d] Simplify $\log_{10} 1000$. **(a)** 1 **(b)** 2 **(c)** 1/2 **(d)** 3 **(e)** 4 **(f)** none of these.

$10^3 = 1000$

(viii) [c] Suppose f is graphed in the figure on the right. Let $g(x) = 4 - f(x+4)$. Where does g take its maximum value on $-4 \le x \le 4$.
(a) $x = -2$ **(b)** $x = -1$ **(c)** $x = 0$ **(d)** $x = 1$ **(e)** $x = 3$ **(f)** none of these.



g's MAXIMUM IS f's MINIMUM, SHIFTED BY 4

(ix) [d] What is the degree 5 coefficient of the polynomial $p(x) = x^2(x+1)(x+3x^3+4x^4)$?
(a) 0 **(b)** 1 **(c)** 2 **(d)** 3 **(e)** 4 **(f)** none of these.

GET x^5 BY $x^2 \cdot 1 \cdot 3x^3$ ONLY

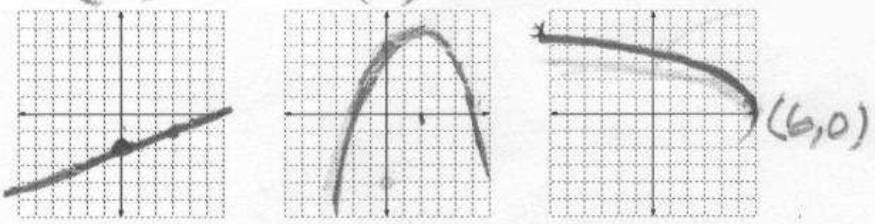
2. Sketch

(i) $y = \frac{1}{3}x - 2$

(ii) $y = -x^2 + 4x + 4$

(iii) $y = \sqrt{6 - x}$

(i) (ii) (iii)



3. Find each of the following limits (or say that it does not exist).

(i) $\lim_{x\to 0} x/|x|$. DOES NOT EXIST SINCE $\frac{x}{|x|} = \begin{cases} 1 & \text{FOR } x > 0 \\ -1 & \text{FOR } x < 0 \end{cases}$

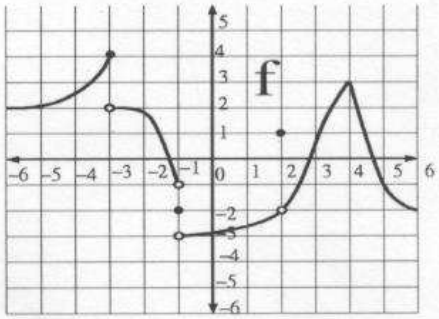
(ii) $\lim_{x\to 0^+} \sqrt{x+4} = \sqrt{4} = 2$

(iii) $\lim_{x\to 2} x^3 + x = 8 + 2 = 10$

(iv) $\lim_{x\to 2}(x^2 - 4)/(x - 2) = \lim_{x\to 2} \frac{(x+2)(x-2)}{x-2} = \lim_{x\to 2}(x+2) = 4$

(v) $\lim_{x\to 1}(x + 1)/(x - 1)$. DOES NOT EXIST.

4. Answer the following questions based on the graph of the function f below. Assume the domain of the function is the interval $0 \le x \le 8$.



(i) What is $f(-3)$? 4

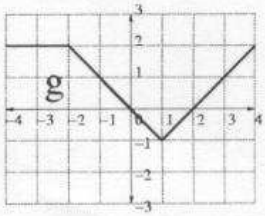
(ii) What is $\lim_{x\to -1^+} f(x)$? -3

(iii) At what point is f continuous from the left but not continuous from the right? -3

(iv) Where is f discontinuous?

-3, -1, AND 2

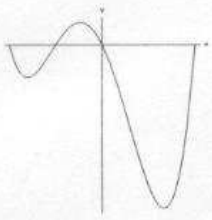
5. Find a formula for the piecewise linear function g graphed below by finding the correct formula for each of its linear segments. Give the intervals on which each formula is valid.

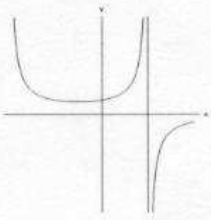


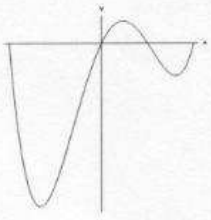
$g(x) = \begin{cases} 2 & \text{FOR } x \le -2 \\ -x & \text{FOR } -2 < x \le 1 \\ x - 2 & \text{FOR } x > 1 \end{cases}$

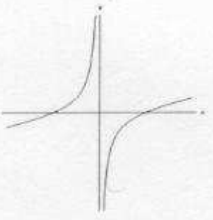
6. Each of the following polynomials and rational functions is graphed below. Match the formulas to the correct graphs (shown on $-2 \le x \le 2$).

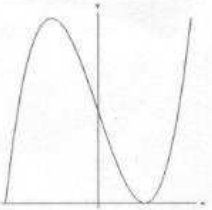
[C] $x(x-1)(x^2-4)$ [E] $(x-1)^2(x+2)$ [F] $\frac{x^2-1}{x^2} + 5 = 6 - \frac{1}{x^2}$ [D] $\frac{x^2-1}{x} = x - \frac{1}{x}$

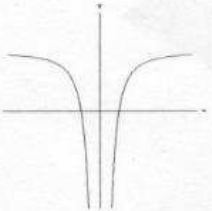
A 

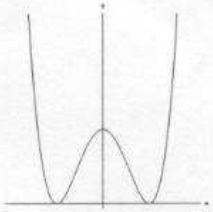
B 

C 

D 

E 

F 

G 

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